# FREE VIBRATION OF LAMINATED COMPOSITE PLATES WITH A CENTRAL HOLE

*A thesis submitted in partial fulfilment of the requirements for the degree of*

**Bachelor of Technology**
**In**
**Civil Engineering**

By

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May 2014

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# CERTIFICATE

This is to certify that the thesis entitled **"FREE VIBRATION OF LAMINATED COMPOSITE PLATES WITH A CENTRAL HOLE"** submitted by **BHABANI SANKAR PADHI** bearing roll no. **110CE0048** of **Civil Engineering Department,** National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date: $6^{th}$ May, 2014

Place: Rourkela

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# ACKNOWLEDGEMENTS

I would like to express my deep sense of gratitude to my supervisor **Prof. A.V. Asha** for giving me this opportunity to work under her. Each and every discussion with her has been very educative. I am very thankful to her for the patience she had with me and for giving me absolute freedom in doing my task. I shall remain indebted to her for her constant support and guidance.

I am thankful to **Prof. N. Roy**, Head of the Department of Civil Engineering, and staff of the Department of Civil Engineering for providing me with necessary facilities for my work.

Truly unbounded words of thanks to my family for their affection, constant encouragement and forbearance. Thanks to the Almighty for the blessing that he has bestowed upon me in all my endeavors.



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# ABSTRACT

Composite materials find a wide range of application, especially in weight sensitive structures like aircraft, spacecraft due to their high strength-to-weight ratio and high stiffness-to-weight ratio. Laminated plates used in these structures are often subjected to dynamic loads. This necessitates the study of buckling and vibrational characteristics of these plates. The presence of holes in the plates makes the analysis complex. Holes are provided for venting, conveyance, maintenance etc. The presence of holes may alter the nature of vibration of the plates. It is, therefore, important to analyze the vibration of laminated composite plates with holes. Also, the effect of various parameters, e.g. boundary condition, aspect ratio, hole-size, number of layers, fiber orientation etc. needs to be analyzed for a safe and stable design of structures.

An orthotropic plate with symmetric fiber orientation was considered for this study. The material properties were fixed. The natural frequencies were computed for different boundary conditions, number of layers, hole-size, aspect ratio, and fiber orientation. The effect of these variables on the nature of vibration is analyzed and discussed. Also, the natural frequencies of a square laminated composite plate with holes of various geometries – circular, square, triangular and hexagonal were computed and compared. ANSYS 13.0 is used for the computation of natural frequencies.

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# CHAPTER-1
# INTRODUCTION

## 1.1 INTRODUCTION

Composite materials are the product of two or more materials combined together on a macroscopic scale to form a useful material. They usually exhibit the best qualities of their constituents and often some qualities that neither constituent possesses. They possess improved strength, stiffness, weight, corrosion resistance, wear resistance, fatigue life etc. Typically, composite materials find applications in weight-sensitive structures such as aircraft and space vehicles, due to their high strength-to-weight and stiffness-to weight ratio.

Laminated fiber-reinforced composites can be viewed as a hybrid class of composites involving both fibrous composites and lamination techniques. Here, each layer consists of fiber-reinforced materials. The fibers in each lamina are oriented in such a manner to give desired strengths and stiffness in the various directions. Thus, the basic advantage with fiber reinforced composite materials is that the strength and stiffness in a specific direction, as per design requirements, can be achieved by proper lamination techniques. Examples of laminated fiber-reinforced composites include Polaris missile cases, fiberglass boat hulls, aircraft wing panels and body sections, tennis rackets, golf club shafts etc.

## 1.2 IMPORTANCE OF PRESENT STUDY

Laminated plates with cut-outs are extensively used in automobiles, aircraft, and space vehicles. Holes of different shapes- circular, square, rectangular, elliptical are used in plates. They serve the purpose of weight reduction, altering resonant frequency, inspection, maintenance, venting, and attachment to other units, for the cables to pass through and so on. It is needed at the bottom plate for passage of liquid in liquid retaining structures. These structures are subjected to undesirable vibration, deflection and rotation during their service life. The presence of cut-outs adds to the complexity of the analysis and design of such structures. The present study deals with the free vibration of laminated composite plates with a single cut-out.

## 1.3 OUTLINE OF PRESENT WORK

An attempt has been made to study the free vibration characteristics of square and rectangular laminated composite plates with cut-outs. For simplification of the analysis, the plate is assumed to be orthotropic and symmetric with respect to the mid-plane. The effect of boundary conditions, hole-size, aspect ratio, number of layers and fiber orientation on the natural frequencies of vibration is studied. First, a single circular hole at the center is considered and then the analysis is further extended to different shapes of cutout at the center of the plate. All the calculations have been done with the finite element package ANSYS 13.0.

This thesis consists of five chapters. The first chapter gives a brief introduction about the importance of the laminated composite plates and application of the present study.

In the second chapter, the relevant literature is presented. All the research papers relevant to this study was critically reviewed and discussed briefly.

The third chapter presents the theoretical formulation. Though, all the computation was carried out using ANSYS, the finite element method and its application in the vibration analysis of laminated composite plates was studied for a deep insight into the subject.

The fourth chapter presents the details of modelling in ANSYS. The relevant steps are clearly discussed.

In chapter 5, the results obtained in the present investigation are tabulated. The effects of various parameters – boundary conditions, number of layers, hole-size, aspect ratio, fiber orientation and hole shape is discussed.

# CHAPTER-2
# LITERATURE REVIEW

## 2.1 INTRODUCTION

Composite plates are typically used in aircraft and automobiles. In most of the structures, these plates are subjected to dynamic loads. Therefore, composite plates are expected to have adequate stiffness to resist failure due to vibration, buckling etc. The frequency of vibration of the plates should be within a certain limit so that it does not affect the function of nearby parts in the structure and does not produce any discomfort. Therefore, it is important to predict the natural frequencies of composite plates to study the behavior of the structure and to avoid resonance of large structures under dynamic loading. The modal analysis can be used as a non-destructive technique for the assessment of stiffness of structures against vibration. The free vibration characteristics of laminated composite plates have been analyzed by many investigators and a number of theoretical and experimental methods have been proposed to predict their natural mode of vibration. The vibrational behavior of plates with cut-outs has also attracted the attention of researchers over the past few decades. The related literature was critically reviewed so as to provide background information on the problems to be considered in the project work and to emphasize the relevance of the present study.

**Rajamani et al.** [5] and [6] considered homogeneous orthotropic composite plates, the laminations of which were assumed to be symmetrical about the mid-plane He investigated the effects of central circular holes and square cut-outs on the natural frequencies of the plate under two end conditions- simply supported (Part 1)and clamped-clamped (Part 2). The effects of cut-out were considered equivalent to an external loading.

**Reddy J.N.** [7] carried investigations on the large amplitude vibration of anisotropic rectangular laminated composite plates. Also, he varied side-to-thickness ratio, aspect ratio and plate side to cut-out side ratio and observed the variation in nature of vibration.

**Lee et al.** [8] studied the simply supported orthotropic rectangular composite plates with central rectangular cut-outs and double square cut-outs. He used Rayleigh principle to predict the natural frequencies, fundamental modes and selected higher modes of the composite plates.

**Bicos et al.** [9] applied finite element method to formulate equations to describe the free damped vibrations of plates and shells. He also developed computer code to calculate the natural frequencies, mode shapes and damping factors of rectangular plates, cylinders and cylindrical panels with different boundary conditions – free, clamped and simply supported. He considered both plates with cutouts and without cutouts.

**Ramakrishna et al.** [11] considered a laminated composite plate with a central circular hole. He developed a computer program for predicting the natural frequencies of vibration of the plate by using a hybrid-stress finite element. Also, he studied the effects of fiber orientation, width-to-thickness ratio, aspect ratio and hole-size on the first four natural frequencies.

**Jwalamalini et al.** [12] considered a simply supported square plate with openings and analyzed its stability under in-plane loading. He used BUCSAP, a Finite Element Program. He took central and square openings for the main study. The tension and compression were assumed as initial pre-stress in the transverse direction before the longitudinal stress was applied.

**Chai Gin Boay** [13] published a paper presenting finite element results on free vibration of laminated composite plates containing a central circular hole considering the aspect ratio and hole-size as variables. The material properties and stacking sequence were kept constant. He considered materials that are typically used in aircraft structural application.

**Sivakumar et al.** [14] investigated the free vibration of laminated composite plates with cut-outs undergoing large oscillations. They used Ritz finite element model and obtained results for plates with cut-outs of various geometries- circle, square, rectangular and ellipse in the large amplitude range.

**Liew et al.** [15] developed a semi analytical procedure to predict natural frequencies of plates with discontinuities in cross-section. He assumed a square element as a basic building element. He used Ritz procedure to extract the frequencies and mode shapes.

**Myung Jo Jhung et al.** [18] analyzed the free vibration of circular plate with eccentric hole. He assumed the plate was submerged in fluid. He developed an analytical method, based on finite

Fourier-Bessel series expansion and Rayleigh-Ritz method. He also varied the hole-size and studied its effects on the vibration characteristics of the plate.

## 2.2 OBJECTIVE OF PRESENT STUDY

Hence after a critical review of literature, it was decided to study the free vibration of laminated composite plates with a central circular hole for various changes in parameters like aspect ratio, fiber orientation, number of layers, etc. The shape of the hole was also varied. The presence of cut-outs adds to the complexity of the analysis and design of structures. The present study deals with the free vibration of laminated composite plates with a single cut-out.

# CHAPTER-3
# THEORY

## 3.1 Governing Differential Equations

The differential equations of motion are obtained by taking a differential element of the panel, as shown in figure 1. This figure shows an element with internal forces like membrane forces $N_x$ ,$N_y$ and $N_{xy}$, shearing forces( $Q_x$ and $Q_y$) and the moment resultants( $M_x,\ M_y\ and\ M_{xy}$).

The governing differential equations of equilibrium for a shear deformable doubly curved panel subjected to external in-plane loading can be expressed as ( Chandrashekhara[9] , Sahu and Dutta) [15]:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = P_1 \frac{\partial^2 u}{\partial t^2} + P_2 \frac{\partial^2 \theta_x}{\partial t^2}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = P_1 \frac{\partial^2 v}{\partial t^2} + P_2 \frac{\partial^2 \theta_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x^0 \frac{\partial^2 w}{\partial x^2} + N_y^0 \frac{\partial^2 w}{\partial y^2} = P_1 \frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = P_1 \frac{\partial^2 \theta_x}{\partial t^2} + P_2 \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = P_1 \frac{\partial^2 \theta_y}{\partial t^2} + P_2 \frac{\partial^2 v}{\partial t^2} \tag{3.1}$$

$N_x^0$ and $N_y^0$ are the external loading in the X and Y directions respectively.

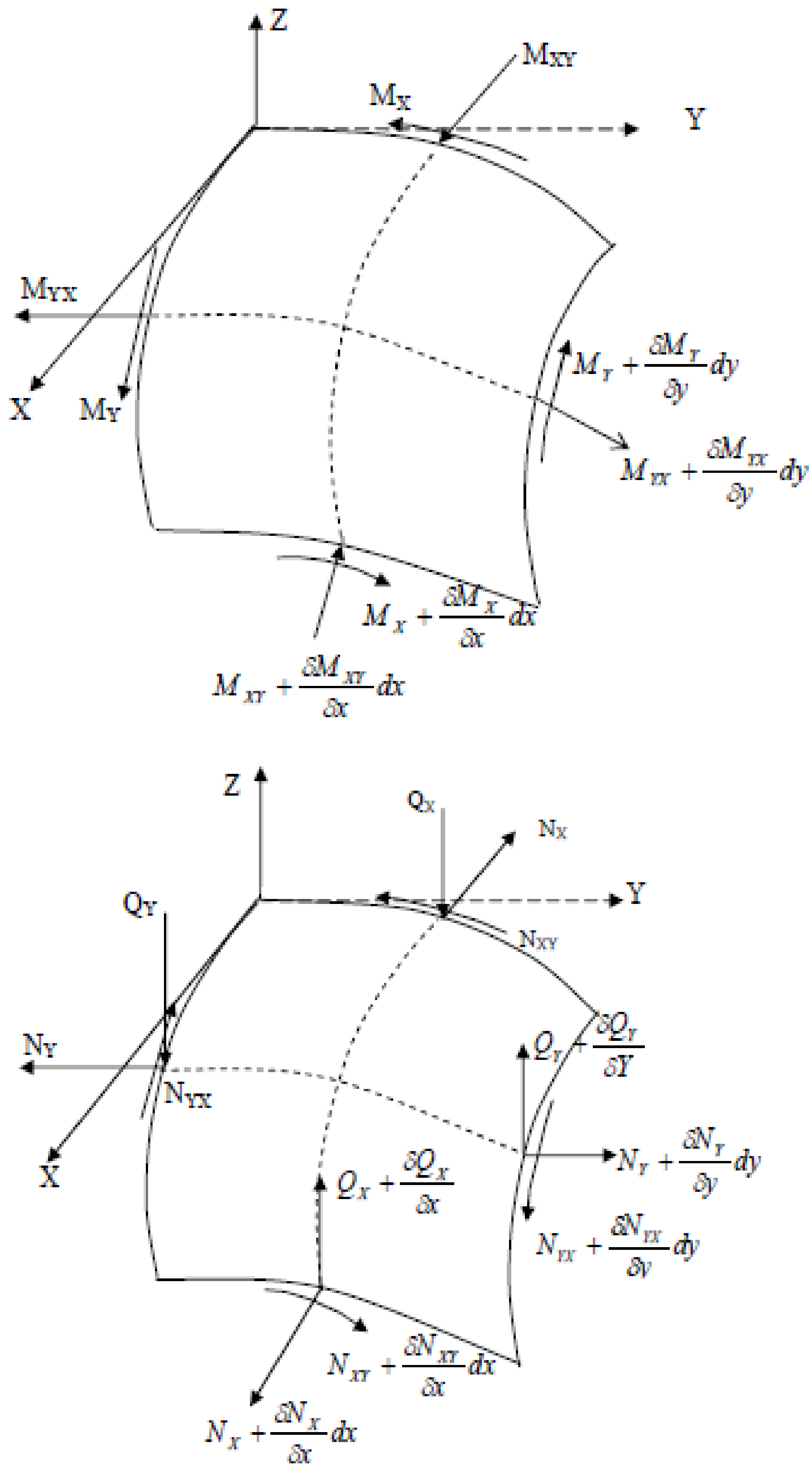



**Figure 3.1: element of a shell panel**

The constants $R_x$, $R_y$ and $R_{xy}$ are the radii of curvature in the x and y directions and the radius of twist.

$$(P_1, P_2, P_3) = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} (\rho)_k (1, z, z^2) dz \tag{3.2}$$

Where, n = number of layers of the laminated composite panel and $(\rho)_k$ = mass density of $k_{th}$ layer from the mid-plane.

In the present study, only flat plates have been analyzed. Hence $R_x$, $R_y$ and $R_{xy}$ are all infinity.

## 3.2 Constitutive Relations:

It is assumed that the composite panel is composed of composite laminates, typically thin layers. The fibers (e.g. graphite, boron, glass) of each lamina are assumed to be parallel and continuous and are embedded in a matrix material (e.g. epoxy resin). Each layer may be regarded on a macroscopic scale as being homogeneous and orthotropic. The laminated fiber reinforced shell is assumed to consist of a number of thin laminates as shown in figure 3.2. The principle material axes are indicated by 1 and 2 and moduli of elasticity of a lamina along these directions are $E_{XX}$ and $E_{YY}$ respectively. The stress strain relationship is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \tag{3.3}$$

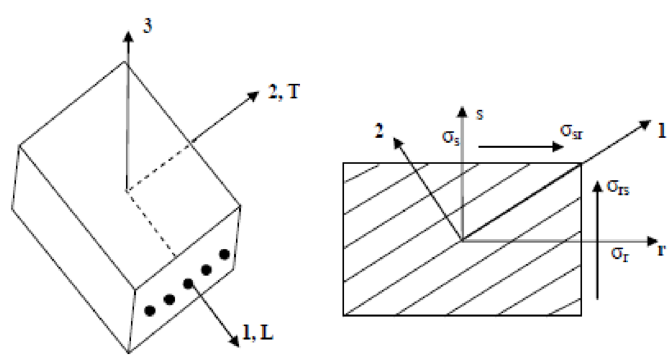



**Figure 3.2: Laminated shell element showing principal axes and laminate directions**

Where

$Q_{11} = \frac{E_{11}}{(1-v_{12}v_{21})}$

$Q_{12} = \frac{E_{11}v_{21}}{(1-v_{12}v_{21})}$

$Q_{21} = \frac{E_{22}}{(1-v_{12}v_{21})}$

$Q_{22} = \frac{E_{22}}{(1-v_{12}v_{21})}$

$Q_{66} = G_{12}$

$Q_{44} = \mathrm{k}G_{13}$

$Q_{55} = kG_{23}$ (3.4)

The on – axis elastic constant matrix corresponding to the fiber direction is given by

$$[\bar{Q}_{ij}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix}$$

If the major and minor Poisson's ratio are $\nu_{12}$ and $\nu_{21}$, then using reciprocal relation one obtains the following well known expression

$$\frac{\vartheta_{12}}{E_{11}} = \frac{\vartheta_{21}}{E_{22}}$$

Standard coordinate transformation is required to obtain the elastic constant matrix for any arbitrary principle axes with which the material principal axes makes an angle $\theta$. Thus the off-axis elastic contant matrix is obtained from the on-axis elastic constant matrix as

$$[\bar{Q}_{ij}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix}$$

$$[\bar{Q}_{ij}] = [T]^T][Q_{ij}][T]$$

Where 'T' is the transformation matrix. After transformation the elastic stiffness coefficients are.

$\bar{Q}_{11} = Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4$

$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4)$

$\bar{Q}_{22} = Q_{11}n^4 + 2(Q_{12} + Q_{66})m^2n^2 + Q_{22}m^4$

$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})nm^3 + (Q_{12} - Q_{22} + 2Q_{66})n^3m$

$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n$

$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2m^2 + Q_{66}(n^4 + m^4)$

The elastic constant matrix corresponding to transverse shear deformation is

$\bar{Q}_{44} = G_{13}m^2 + G_{23}n^2$

$\bar{Q}_{45} = (G_{13} - G_{23})mn$

$\bar{Q}_{55} = G_{13}n^2 + G_{23}m^2$

Where m = cos$\theta$ and n = sin$\theta$

The stress strain relations are

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \tag{3.5}$$

The forces and moment resultants are obtained by integration through the thickness h for stresses as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_Y \\ M_{xY} \\ Q_x \\ Q_Y \end{bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_x \\ \tau_{xy} \\ \sigma_x z \\ \sigma_{yz} \\ \tau_{xy} z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz$$

Where $\sigma_x$, $\sigma_y$ are the normal stresses along X and Y direction, $\tau_{xy}$,$\tau_{xz}$ and $\tau_{yz}$ are shear stresses in xy , xz and yz planes respectively.

Considering only in-plane deformation, the constitutive relation for the initial plane stress analysis is

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{31} & A_{32} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

The constitutive relationships for bending transverse shear of a doubly curved shell becomes

$$\begin{Bmatrix} N_x \\ N_P \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{112} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}$$

This can also be stated as

$$\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_m \end{Bmatrix}$$

Or $\{F\} = [D]\{\varepsilon\}$

Where$A_{ij}$,$B_{ij}$,$D_{ij}$and $S_{ij}$ are the extensional , bending-stretching coupling, bending and transverse shear stiffness. They may be defined as:

$$A_{ij} = \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^3 - z_{k-1}^3) \,;\, i,j = 1,2,6$$

$$S_{ij} = k \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1}) \,;\, i,j = 4,5$$

And k is the transverse shear correction factor.

# CHAPTER-4
# MODELLING USING ANSYS 13.0

## 4.1 INTRODUCTION

ANSYS (acronym for Analysis System) is a general purpose Finite Element Analysis (FEA) program that solves a vast area of solid and structural mechanics problems in geometrically complicated regions. In the present work, ANSYS 13.0 is used to model the plate, to compute natural frequencies and to plot deformed shapes.

In the following sub-sections, details of the modelling are presented. First, some terms related to this topic are explained. Then, the procedure of modelling is presented.

## 4.2 TERMINOLOGIES

Shell 281: Shell 281 is used as an element type for thin to moderately thick shell structures. It has eight nodes with six degrees of freedom at each node: translations in the x, y, and z axes, and rotations about the x, y, and z-axes. It follows first order shear deformation theory. The geometry, nodes and co-ordinate system of a shell element is shown below [18].

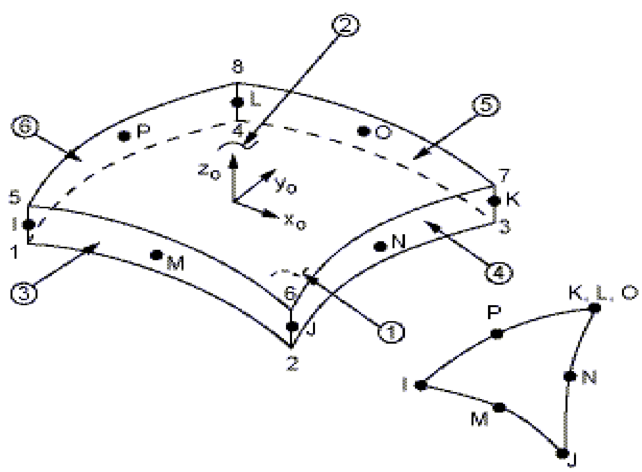


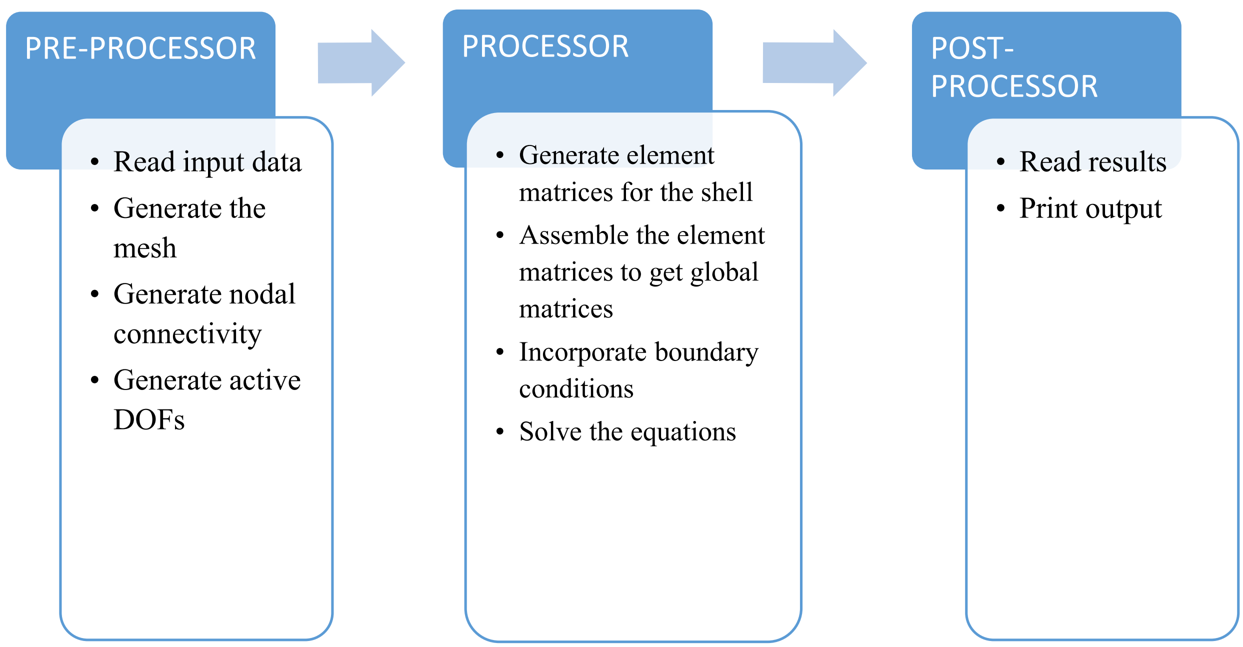

**Figure 4.1: A SHELL 281 Element**

## Modal Analysis:

Modal analysis in ANSYS is a linear analysis. Several mode extraction methods, e.g. Block Lanczos, Supernode, PCG Lanczos, reduced, unsymmetric, damped, and QR damped are available. Block Lanczos is used in the present analysis. It is used for large symmetric eigenvalue problems. This method uses the sparse matrix solver. Similarly all the methods have their own limitations. For more information on the methods of mode extraction, the reader may follow ANSYS Technical Guide[18].

### 4.3 PROCEDURE

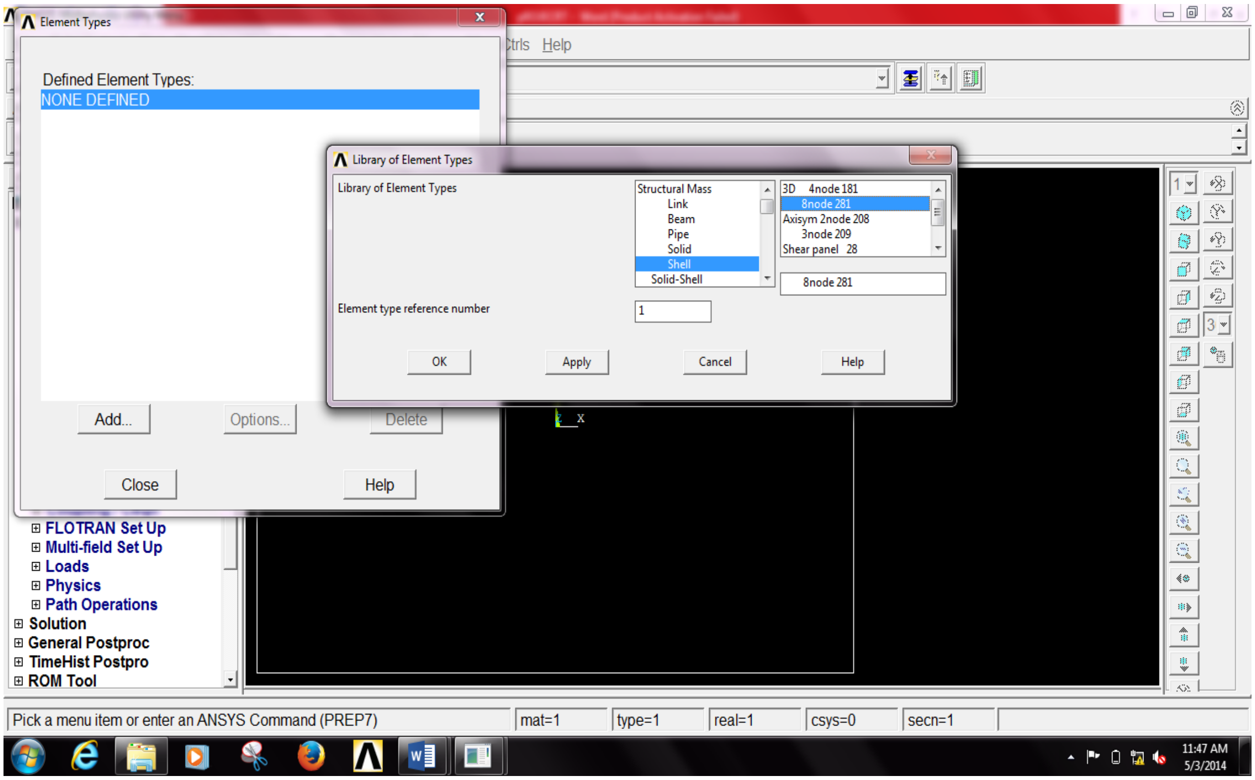
The following flow chart shows an overview of the steps to be followed in ANSYS.



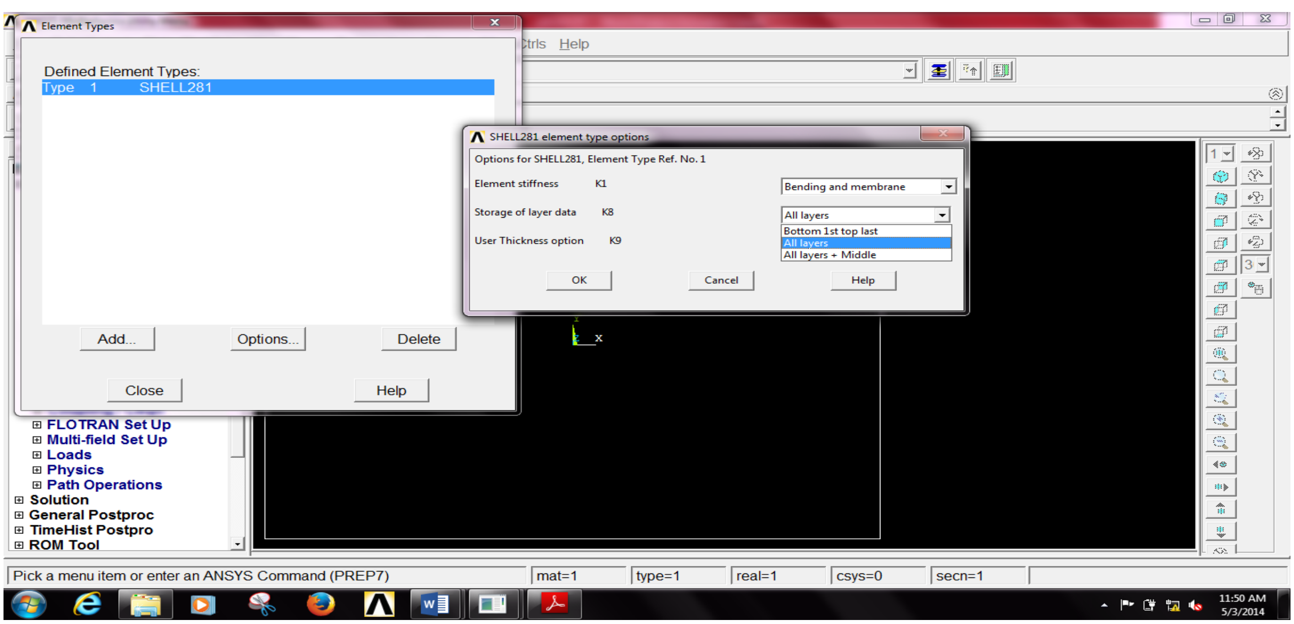
### 4.3.1 PREPROCESSOR

It involves the following steps. The following tabs can be found in Utility Menu > Preprocessor.

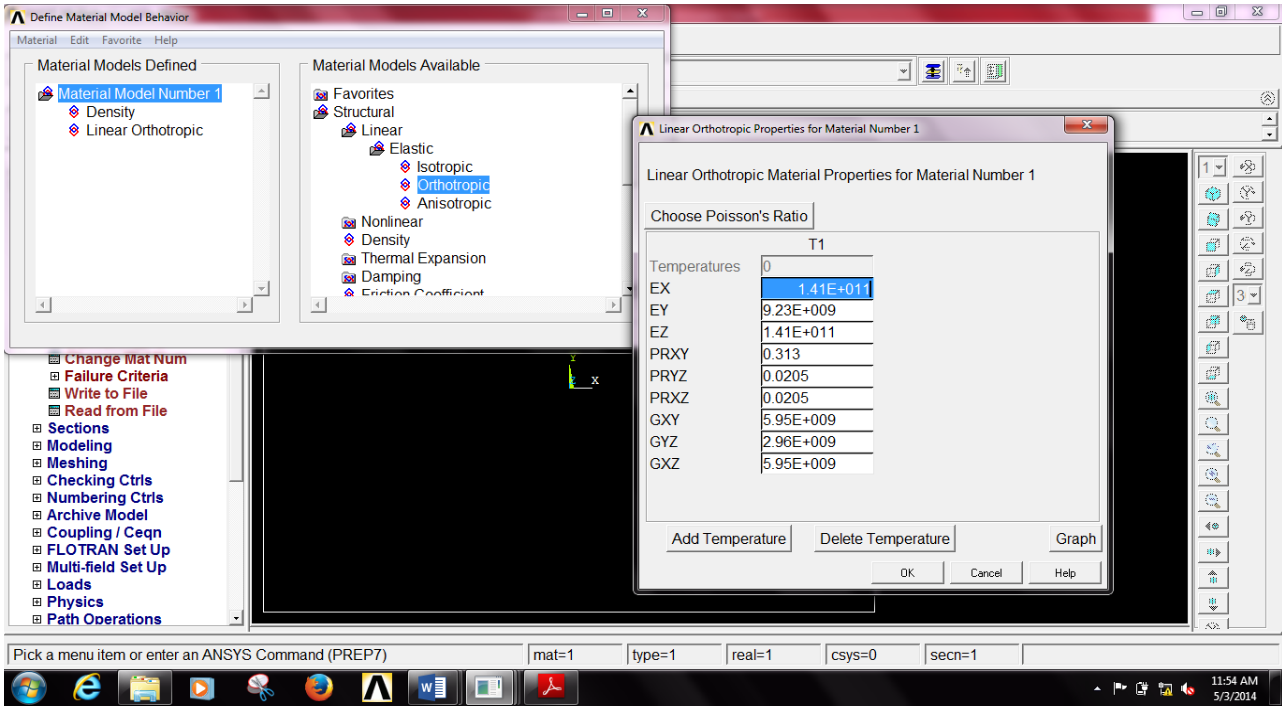
1. Element Type > Add > Structural Mass > Shell > 8 node 281



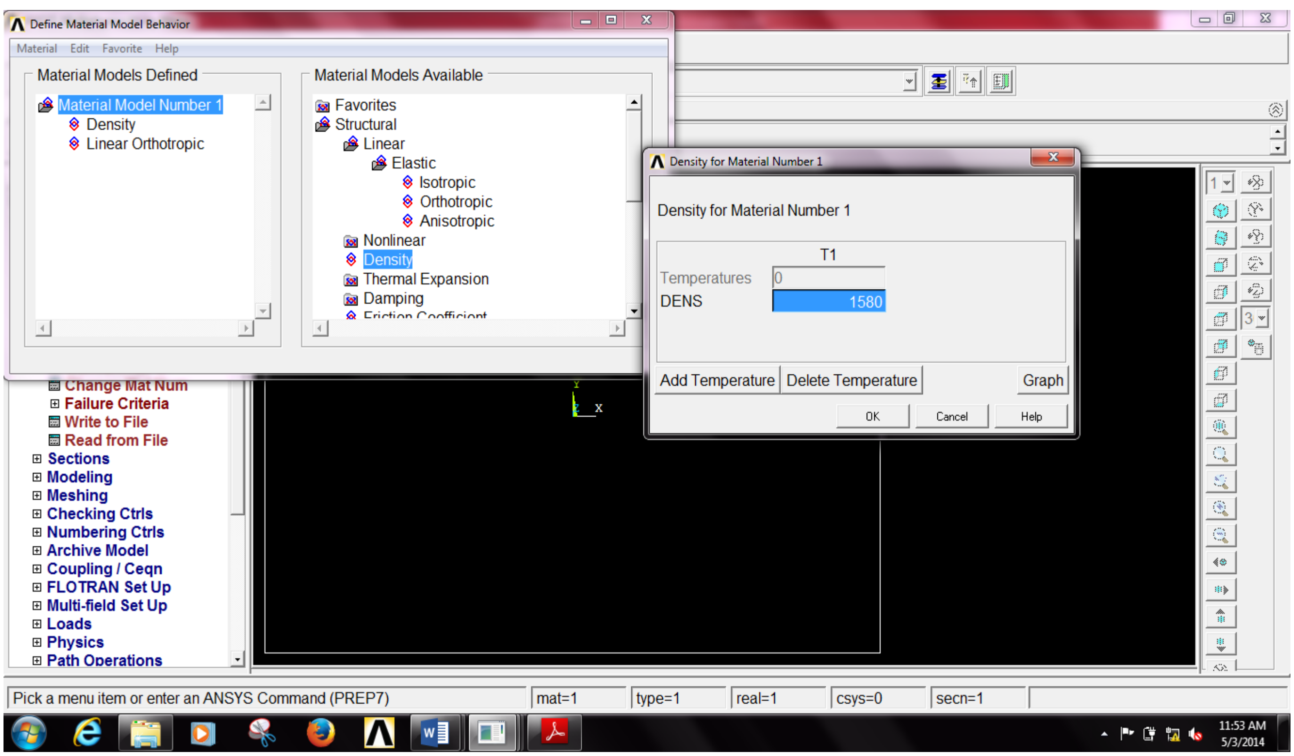

1. Select Shell 281 > Options > Storage of layer data – All layers



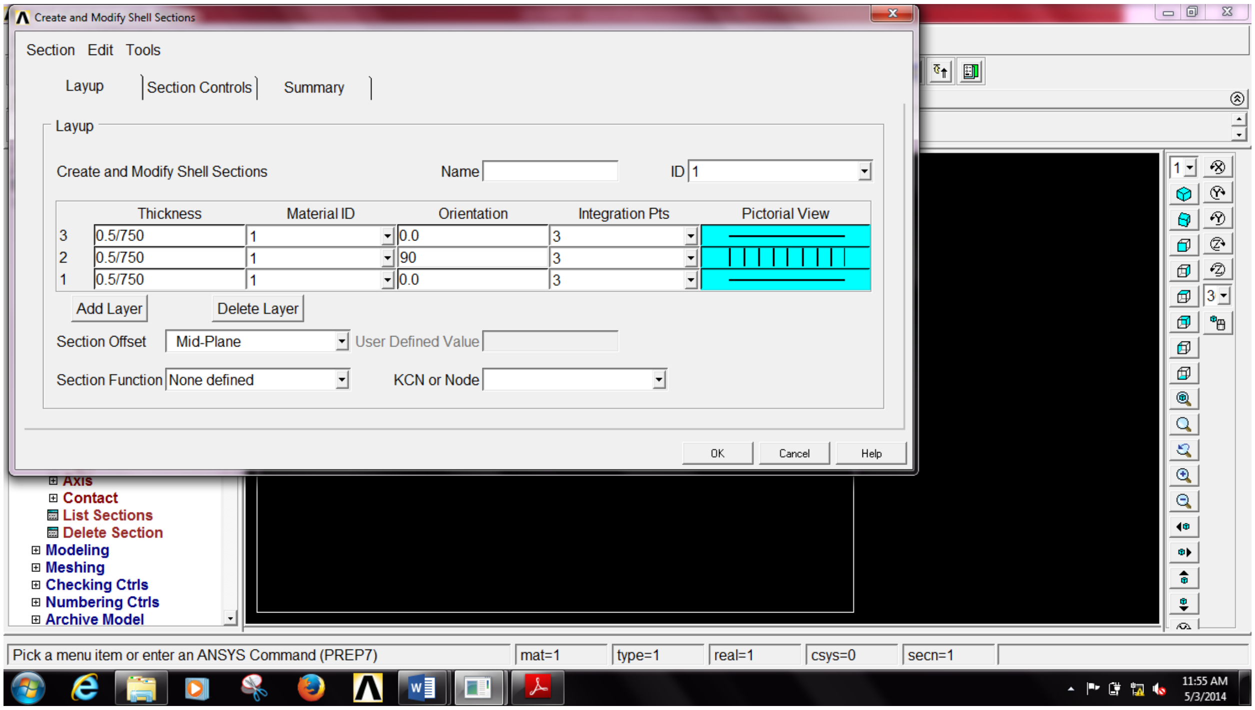
1. Material models > Material properties > Structural > Linear > Elastic > Orthotropic – Enter the linear orthotropic properties



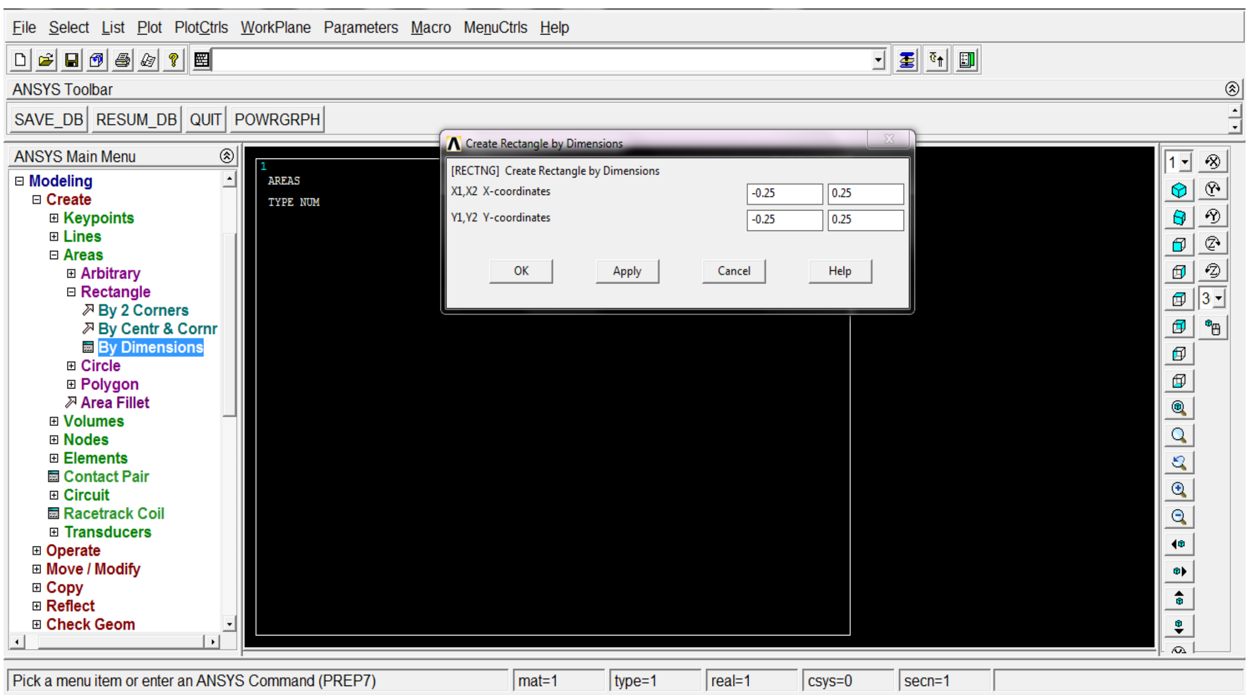

2. Material models > Density > Enter the value of density



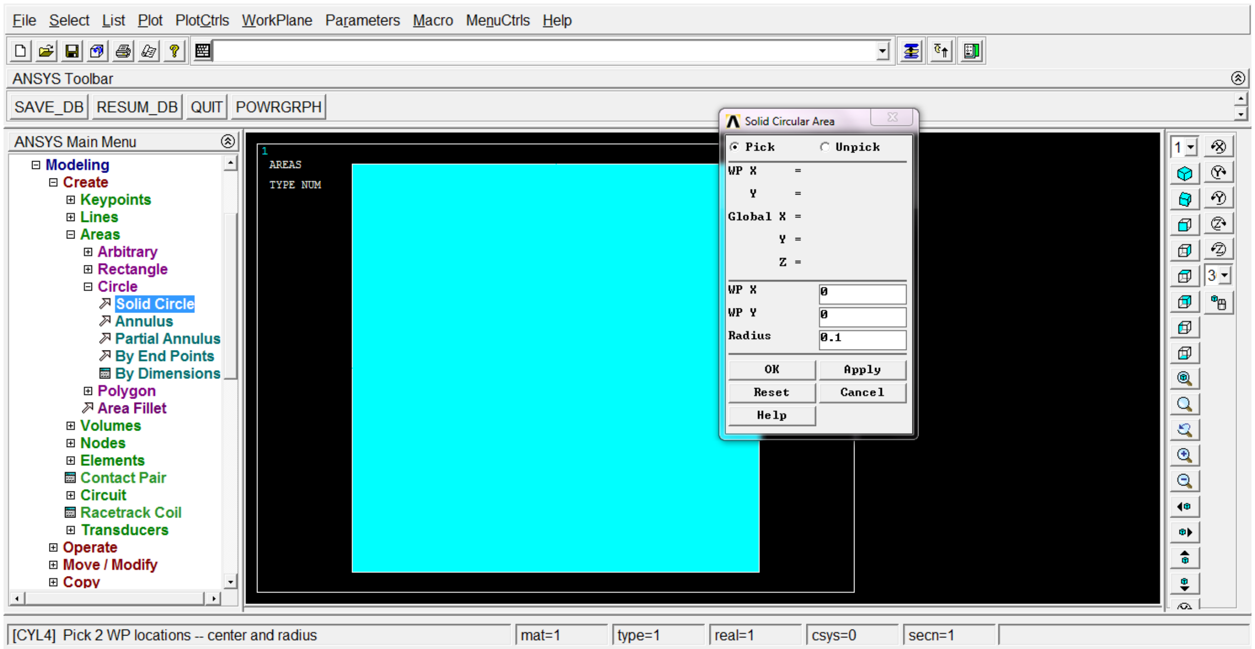
1. Sections > Shell > Lay-up > Add/Edit > Enter thickness and fiber orientation of each layer



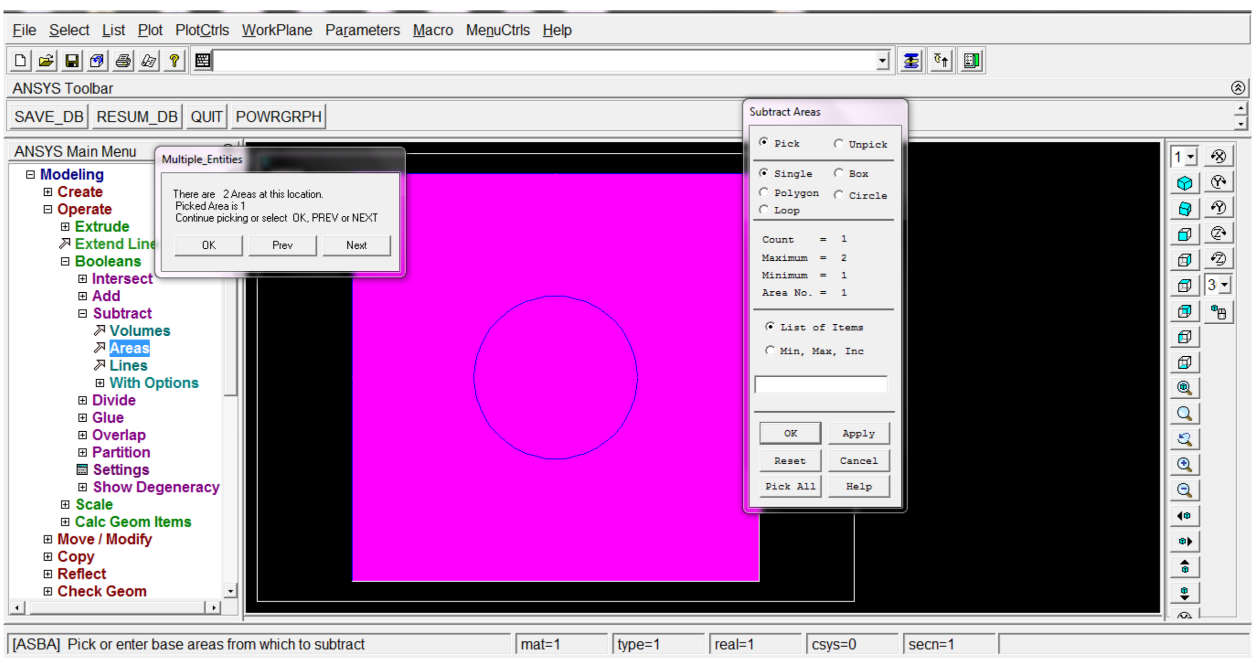

2. Modelling > Create > Areas > Rectangle > By dimension - Enter the co-ordinates of the corner points (There are 3 possible ways in which rectangle can be created)



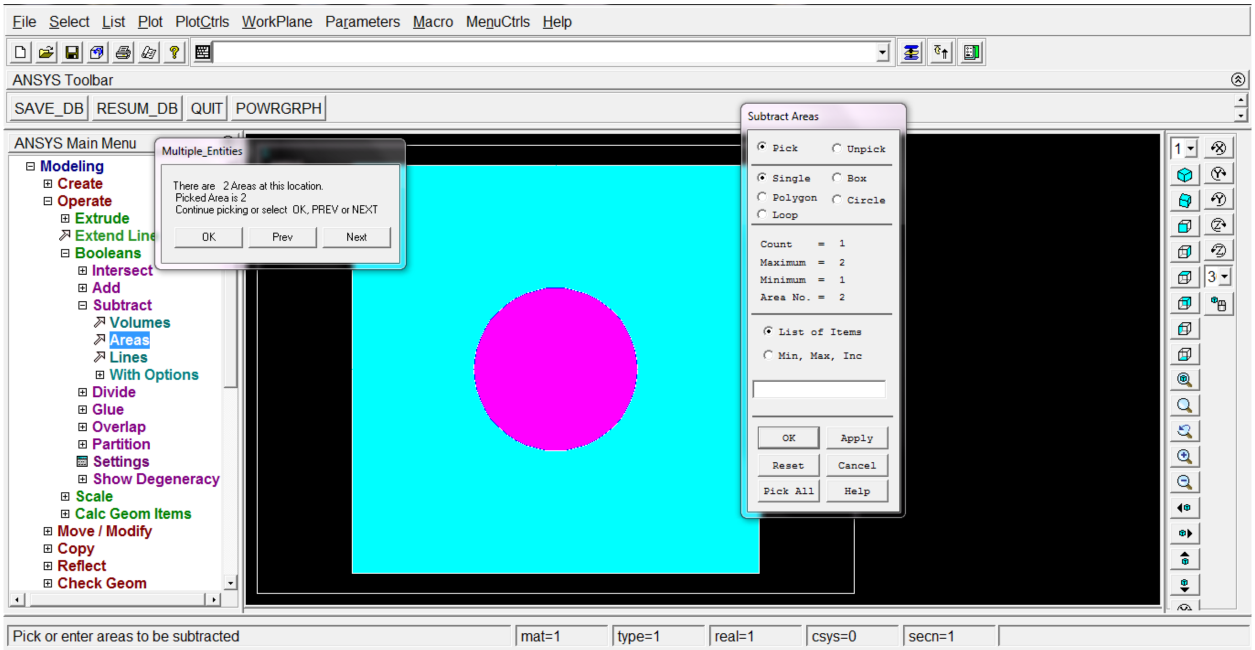
1. Modelling > Create > Areas > Circle > Solid Circle – Enter the co-ordinates of the center and the radius



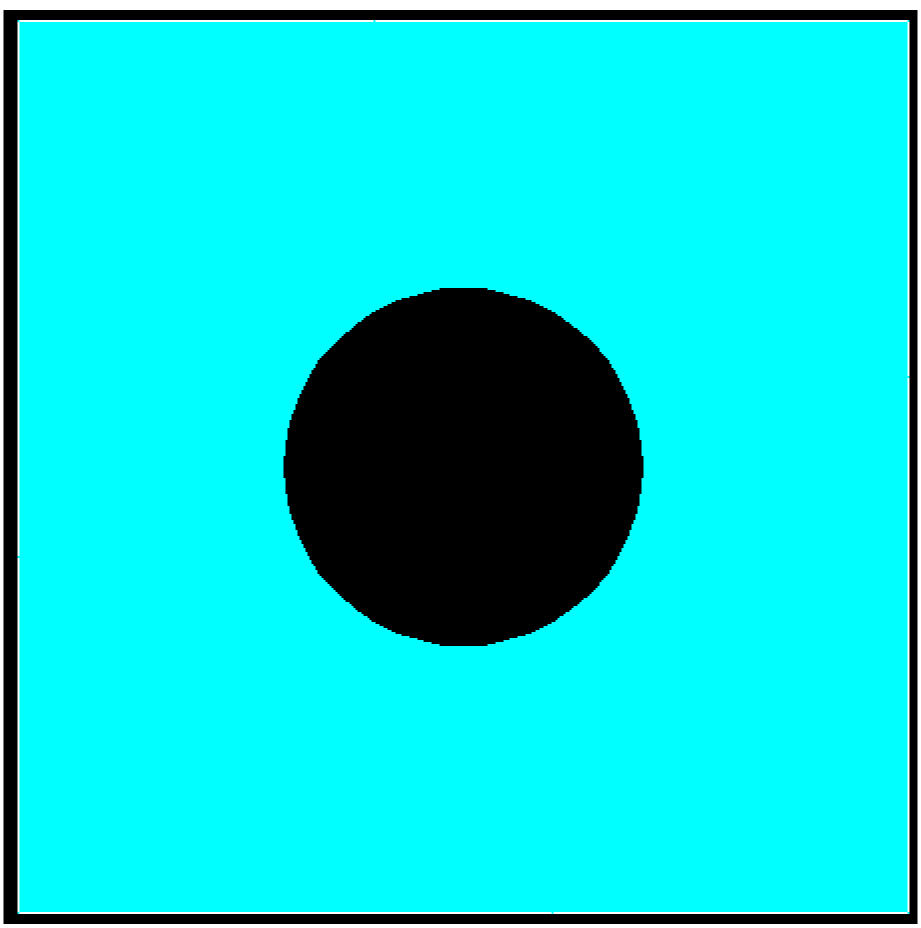

2. Modelling > Operate > Booleans > Subtract > Areas – First select the area to be subtracted from. Click Ok.



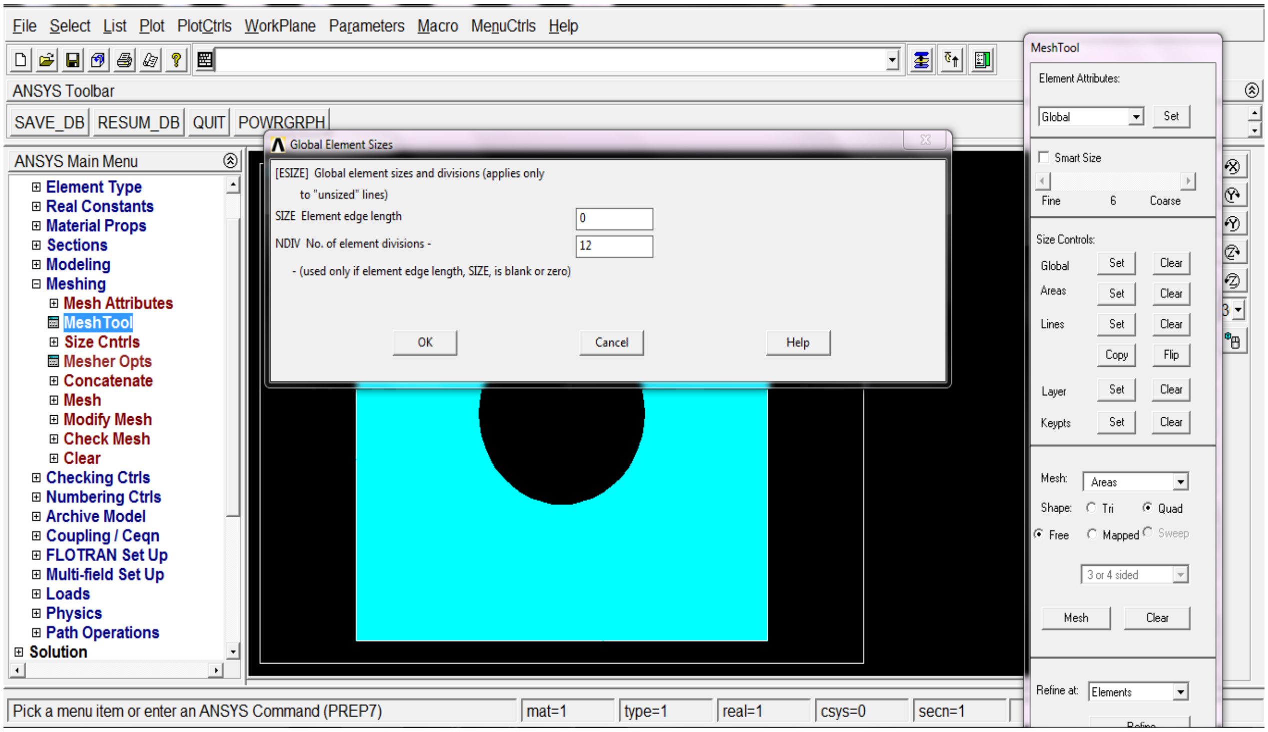

Then, select the area to be subtracted. Click Ok.



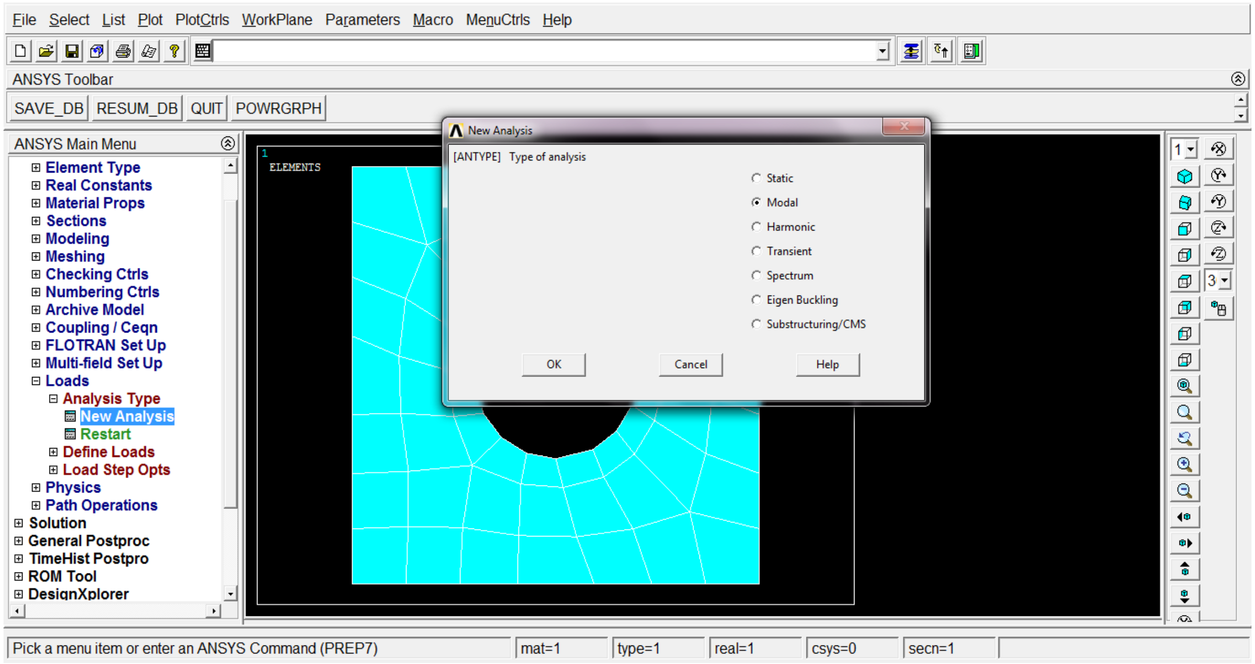

The plate with a central circular hole is shown below.



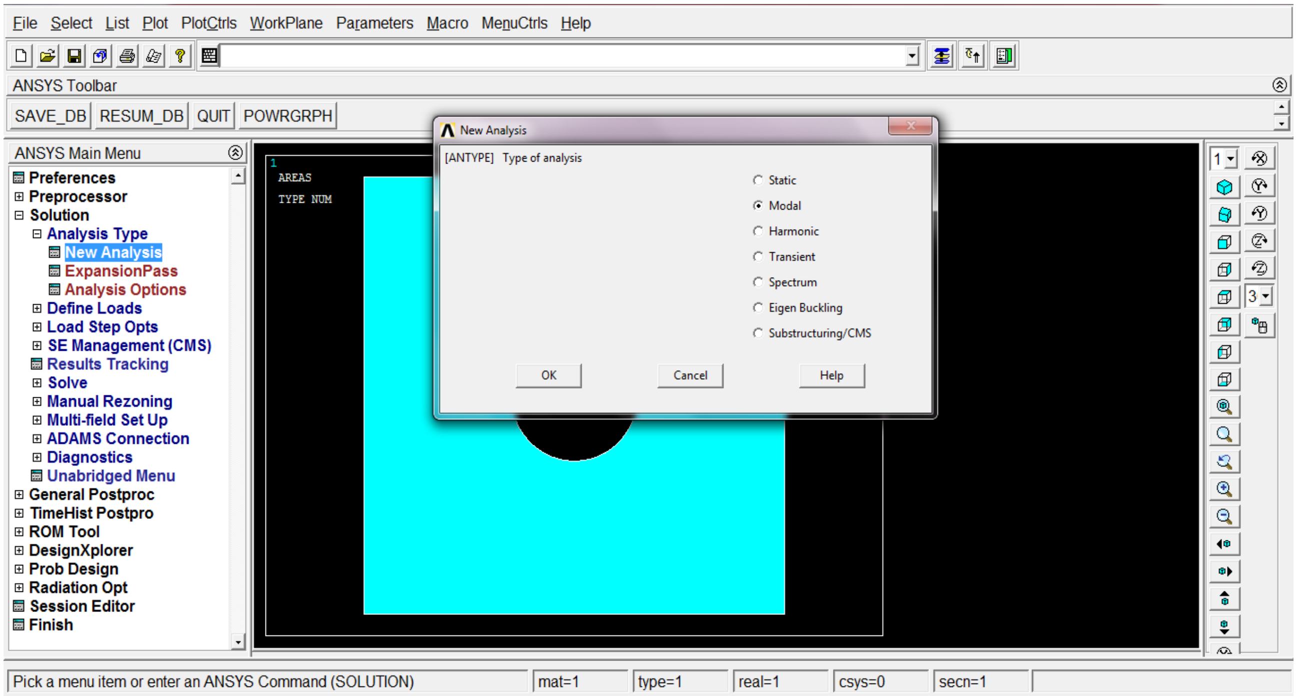
1. Meshing > Mesh Tools > Size controls : Global > Set > Enter number of divisions



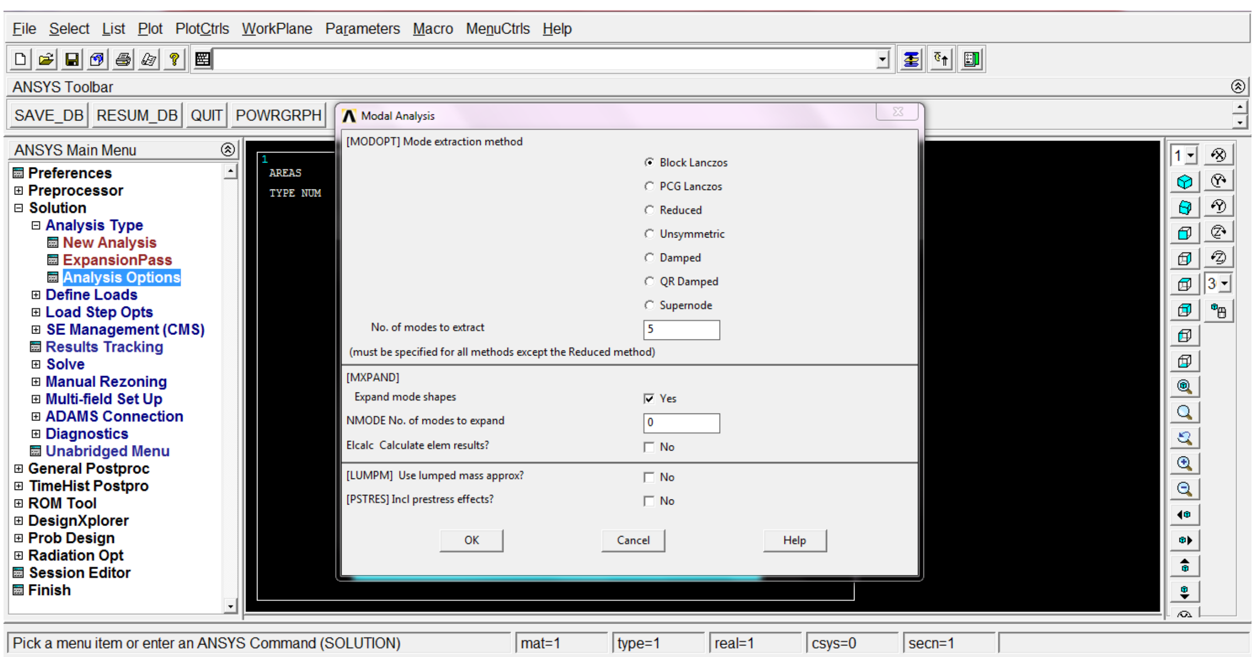
2. Loads > Define loads > Apply > Structural > Displacement > On lines > Pick lines > Apply displacement on lines



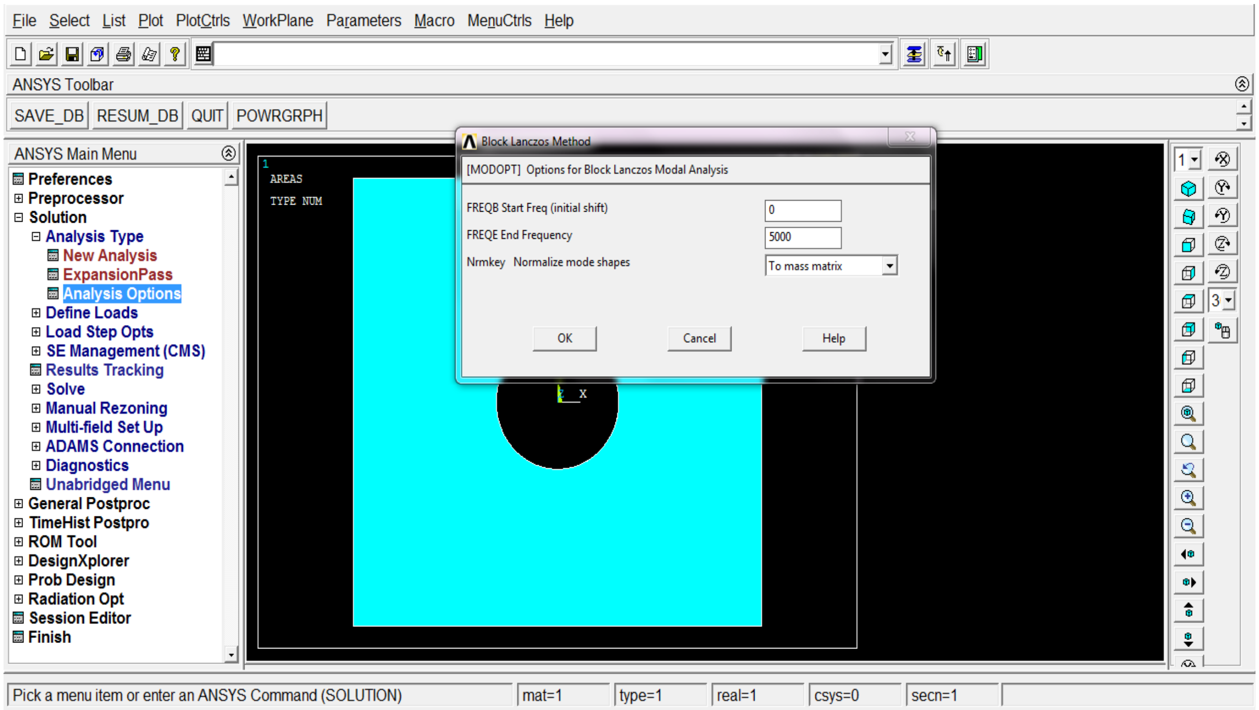
11. Loads > Analysis type > New analysis > Type of analysis : Modal




1. Loads > Analysis type > Analysis options > Mode-extraction method – Block Lanczos (select any one of the available options) > Number of modes to extract-10 (enter a number > 0)



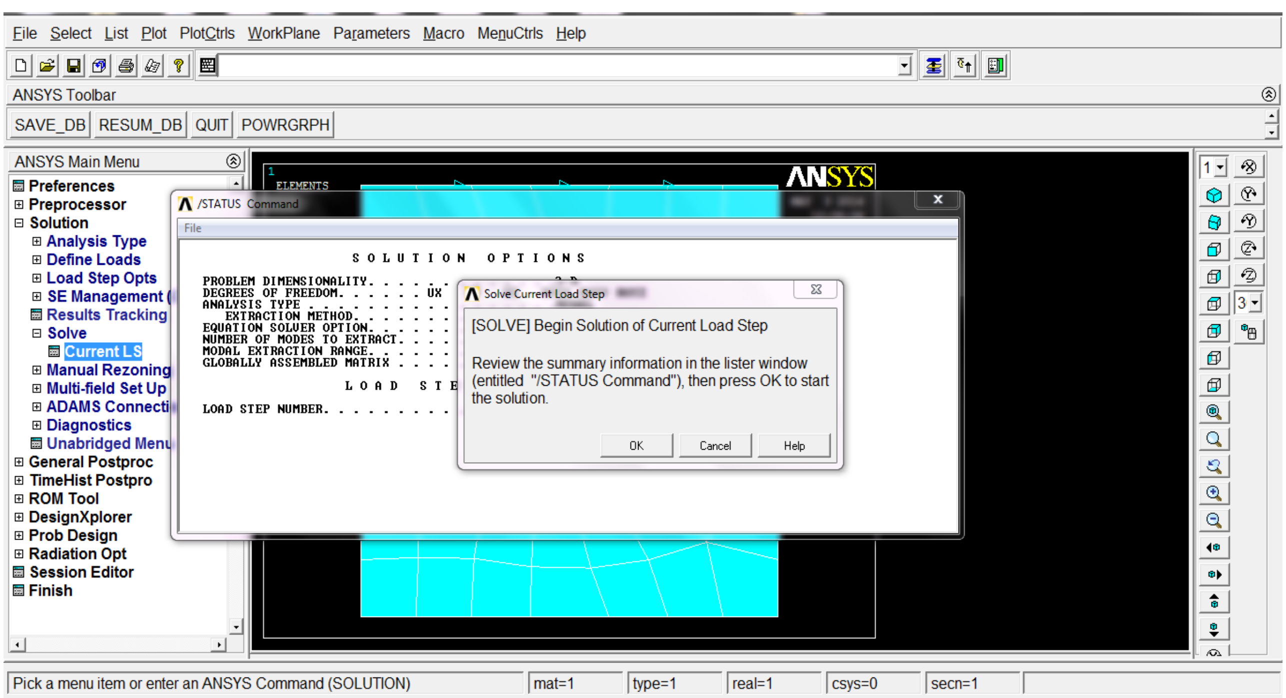

Then, Enter Start and End frequencies, as shown below




## 4.3.2 PROCESSOR

It is the solution stage. The program solves the concerning equations using the input data provided in preprocessor stage.

1. Utility Menu > Solution > Solve > Current LS



### 4.3.3 POSTPROCESSOR

Here, the results can be viewed, mode shapes can be plotted. A number of forms of graphical representation of the results are available, including deformed shape, deformed + undeformed shape, contour plot, vector plot etc.

1. Utility Menu > General Postproc > Result Summary
2. Utility Menu > General Postproc > Plot Results > Deformed shape – Select the desired shape

# CHAPTER-5

# RESULTS AND DISCUSSION

An attempt has been made to study the free vibration characteristics of square and rectangular laminated composite plates with cut-outs. For simplification of the analysis, the plate is assumed to be orthotropic and symmetric with respect to mid-plane. The effect of hole-size, aspect ratio, number of layers and fiber orientation on the natural frequencies of vibration is studied. First, a single circular hole at the center is considered and then the analysis is further extended with holes of various shapes at the center. As mentioned earlier, the studies have been done with the finite element package ANSYS 13.0.

## 5.1 CONVERGENCE AND VALIDATION STUDY:

In this section, the convergence study was done for free vibration analysis of laminated composite plates without hole and with hole for determining mesh size and the results are compared with available published literature obtained using different numerical methods.

### 5.1.1 Vibration analysis of laminated composite plate without hole

A simply supported square cross ply (0°/90°/0°) plate is considered. The side of the plate is 1 m. The natural frequencies are computed for two values of side-to-thickness ratio - $\frac{a}{h}=10$ and $\frac{a}{h}=100$ . The mesh divisions are varied from 4 x 4 till the values converge. Finally, the results are compared with the results obtained by Liu at el [17].

Material Properties:

$$\rho = 1643\,\frac{\text{kg}}{\text{m}^3}$$

$$\mu_{XY} = 0.25 \qquad E_{XX} = 1.90\times10^{11}\,\frac{\text{N}}{\text{m}^2} \qquad G_{XY} = 3.8\times10^{9}\,\frac{\text{N}}{\text{m}^2}$$

$$\mu_{YZ} = 0.01 \qquad E_{YY} = 7.6\times10^{9}\,\frac{\text{N}}{\text{m}^2} \qquad G_{YZ} = 1.52\times10^{9}\,\frac{\text{N}}{\text{m}^2}$$

$$\mu_{XZ} = 0.01 \qquad E_{ZZ} = 1.90\times10^{11}\,\frac{\text{N}}{\text{m}^2} \qquad G_{XZ} = 3.8\times10^{9}\,\frac{\text{N}}{\text{m}^2}$$

**Table 1: Convergence study for laminated composite plate with** $\frac{a}{h} = 10$

| Mesh Division | 1st mode | 2nd mode | 3rd mode |
|---|---|---|---|
| 4x4 | 11.306 | 18.089 | 31.171 |
| 8x8 | 11.297 | 17.938 | 30.283 |
| 12x12 | 11.297 | 17.930 | 30.201 |
| Result Liu et al. [17] | 11.455 | 18.333 | 31.141 |

**Table 2: Convergence study for laminated composite plate with** $\frac{a}{h} = 100$

| Mesh Division | 1st mode | 2nd mode | 3rd mode |
|---|---|---|---|
| 4x4 | 15.145 | 23.200 | 43.687 |
| 8x8 | 15.147 | 22.769 | 40.242 |
| 12x12 | 15.148 | 22.757 | 40.111 |
| Result Liu et al. [17] | 15.127 | 22.658 | 39.644 |

Thus, the mesh-divisions are finalized as 12x12.

The results as obtained above are found to be in agreement with the results obtained by Liu et al [17]. A comparison of the results is presented in Table 3. Liu et al[17] used a radial point interpolation method (RPIM) for the analysis. As seen from Table 3, the results obtained using present ANSYS formulation are in good agreement with those of Liu et al.

**Table 3: Comparison of normalized frequencies for laminated composite plate without hole**

| Property index | Mode 1 | | Mode 2 | | Mode 3 | |
|---|---|---|---|---|---|---|
| | Present study | RPIM Liu et al. [17] | Present study | RPIM Liu et al. [17] | Present study | RPIM Liu et al. [17] |
| $\frac{a}{h}=10$ | 11.297 | 11.455 | 17.930 | 18.333 | 30.201 | 31.141 |
| $\frac{a}{h}=100$ | 15.148 | 15.127 | 22.757 | 22.658 | 40.111 | 39.644 |

5.1.2 Vibration analysis of laminated composite plate with hole

The natural frequencies of a laminated composite plate with a central circular cut-out was also determined and was found to be in close agreement with the results obtained by Ramakrishna et al [11]. The results are given in Table 4. The dimensions and material properties of the plate are as given below:

Dimension:

22.86 cm side,

23.876 m thickness

$$\frac{\text{hole dia}}{\text{side}}=0.1$$

Material Properties:

$$\rho=1380.3\frac{\text{kg}}{\text{m}^3}$$

$\mu_{XY}=0.35$ $\quad E_{XX}=14.341\text{GPa}$ $\quad G_{XY}=1.862\text{ GPa}$

$\mu_{XZ}=0.35$ $\quad E_{YY}=5.102\text{GPa}$ $\quad G_{YZ}=0.689\text{GPa}$

$\mu_{YZ}=0.35$ $\quad E_{ZZ}=14.431\text{GPa}$ $\quad G_{XZ}=1.862\text{GPa}$

**Table 4: Comparison of first four natural frequencies for laminated composite plate with central circular hole**

| Mode | 1st | 2nd | 3rd | 4th |
|---|---|---|---|---|
| Present Study | 205.83 | 378.63 | 383.23 | 576.24 |
| Ramakrishna et al. [11] | 200 | 380 | - | 575 |

Thus, as observed from Table 4, the results are in agreement with those of Ramakrishna et al [11].

## 5.2 PRESENT STUDY

A laminated composite plate with a central circular hole is considered and the natural frequencies are computed. Also, the effect of boundary conditions, number of layers, aspect ratio (length-to-width ratio), hole-size and fiber orientation on the free vibration characteristics is analyzed. The results are presented through following sub-sections.

1. Effect of Boundary Conditions
2. Effect of number of Layers
3. Effect of Hole-size
4. Effect of Aspect-Ratio
5. Effect of fiber-orientation
6. Effect of Hole-shape

Plate dimension –

$\mathrm{a} = 0.5\,\mathrm{m}$ a/b = 1( unless otherwise mentioned)

$\frac{\mathrm{b}}{\mathrm{h}} = 250$ b and D are variables.

The material properties to be used in the analysis are presented below.

$$\rho = 1580 \frac{\mathrm{kg}}{\mathrm{m}^3}$$

$\mu_{XZ} = 0.313$ $E_{XX} = 141\mathrm{GPa}$ $G_{XY} = 5.95\,\mathrm{GPa}$

$\mu_{XZ} = 0.0205$ $E_{YY} = 9.23\,\mathrm{GPa}$ $G_{YZ} = 2.96\,\mathrm{GPa}$

$\mu_{YZ} = 0.0205$ $E_{ZZ} = 141\mathrm{GPa}$ $G_{XZ} = 5.95\,\mathrm{GPa}$

### 5.2.1 Effect of Boundary conditions

A laminated cross ply square plate (0°/90°/0°) with three different boundary conditions as given below is studied

1. All edges simply supported (SSSS)
2. All edges clamped (CCCC)
3. Cantilever plate (CFFF)

Here, $\left(\frac{D}{a}\right)=0.1$ and $\left(\frac{a}{b}\right)=1$.

The first five natural frequencies are presented in Table 5.

**Table 5: First five natural frequencies (Hz) for a square cross-ply (0°/90°/0°) with central circular hole for three different boundary conditions**

| Mode | SSSS | CCCC | CFFF |
|---|---|---|---|
| 1 | 28.53 | 70.35 | 6.82 |
| 2 | 92.10 | 148.39 | 14.02 |
| 3 | 97.30 | 171.39 | 42.24 |
| 4 | 137.66 | 223.45 | 46.78 |
| 5 | 166.80 | 252.75 | 58.35 |

From the above table, it can be observed that the natural frequency of vibration is highest for CCCC plate, whereas it is lowest for CFFF plate. Simply supported plates show intermediate values. The following graph in Figure 5.1 depicts the variation in natural frequencies with boundary conditions.

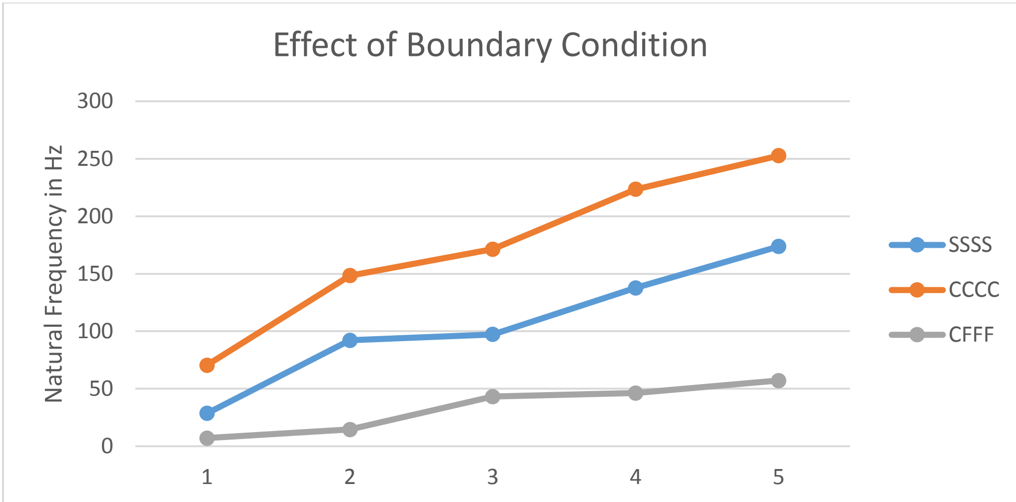



**Fig. 5.1: Variation of natural frequencies (Hz) for a square cross-ply plate with central circular hole for three different boundary conditions**

The mode shapes corresponding to the lowest frequency for the above boundary conditions are shown in Figure 5.2

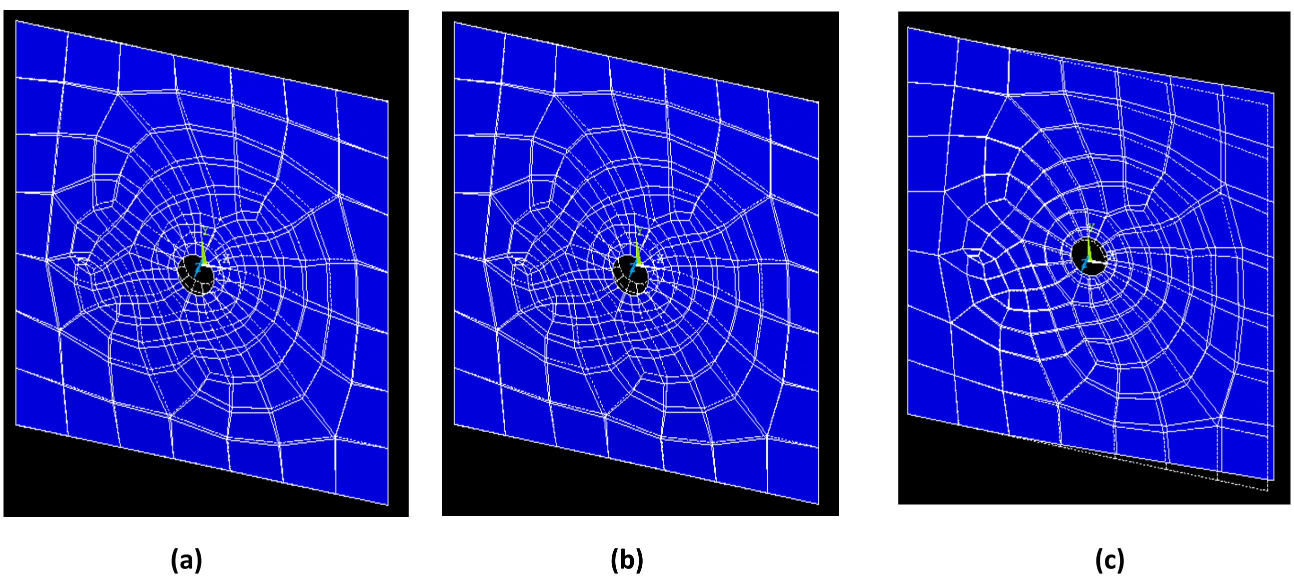



**Fig 5.2: Mode shapes for a square cross-ply (0°/90°/0°) plate with a central circular hole for the boundary condition (a) SSSS, (b) CCCC, (c) CFFF**

### 5.2.2 Effect of number of layers

The above results were obtained for a symmetric three layer cross-ply plate. In this section, the variation of natural frequencies with variation in number of layers of laminates was studied for a four layer and eight layer cross-ply plate keeping thickness of the square plate constant.

Here, $\left(\frac{D}{a}\right) = 0.1$ and $\left(\frac{a}{b}\right) = 1$.

The following table presents the first five natural frequencies of the plate with three, four and eight numbers of layers and for three different boundary conditions.

**Table 6: First five natural frequencies (Hz) for the boundary condition SSSS and varying number of layers**

| Mode | 3 Layers (0°/90°/0°) | 4 Layers (0°/90°/90°/0°) | 8 Layers (0°/90°/0°/90°/0°/90°/0°/90°) |
|---|---|---|---|
| 1 | 28.53 | 37.21 | 36.82 |
| 2 | 92.10 | 94.00 | 96.08 |
| 3 | 97.30 | 94.76 | 97.41 |
| 4 | 137.66 | 151.46 | 155.53 |
| 5 | 173.80 | 200.39 | 204.95 |

**Table 7: First five natural frequencies (Hz) for the boundary condition CCCC and varying number of layers**

| Mode | 3 Layers (0°/90°/0°) | 4 Layers (0°/90°/90°/0°) | 8 Layers (0°/90°/0°/90°/0°/90°/0°/90°) |
|---|---|---|---|
| 1 | 75.99 | 84.50 | 89.57 |
| 2 | 136.24 | 149.45 | 157.63 |
| 3 | 145.56 | 149.79 | 161.63 |
| 4 | 226.10 | 233.18 | 247.05 |
| 5 | 259.57 | 277.99 | 298.02 |

**Table 8: First five natural frequencies (Hz) for a CFFF square laminate with varying number of layers**

| Mode | 3 Layers (0°/90°/0°) | 4 Layers (0°/90°/90°/0°) | 8 Layers (0°/90°/0°/90°/0°/90°/0°/90°) |
|---|---|---|---|
| 1 | 7.02 | 8.04 | 8.41 |
| 2 | 14.52 | 15.73 | 16.28 |
| 3 | 43.14 | 47.25 | 48.67 |
| 4 | 46.26 | 55.90 | 56.15 |
| 5 | 57.02 | 61.53 | 63.13 |

From the above three tables, it can be observed that with the increase in number of layers, there is an increase in the frequency of vibration. The variation in frequencies can be best understood from the following graphs.

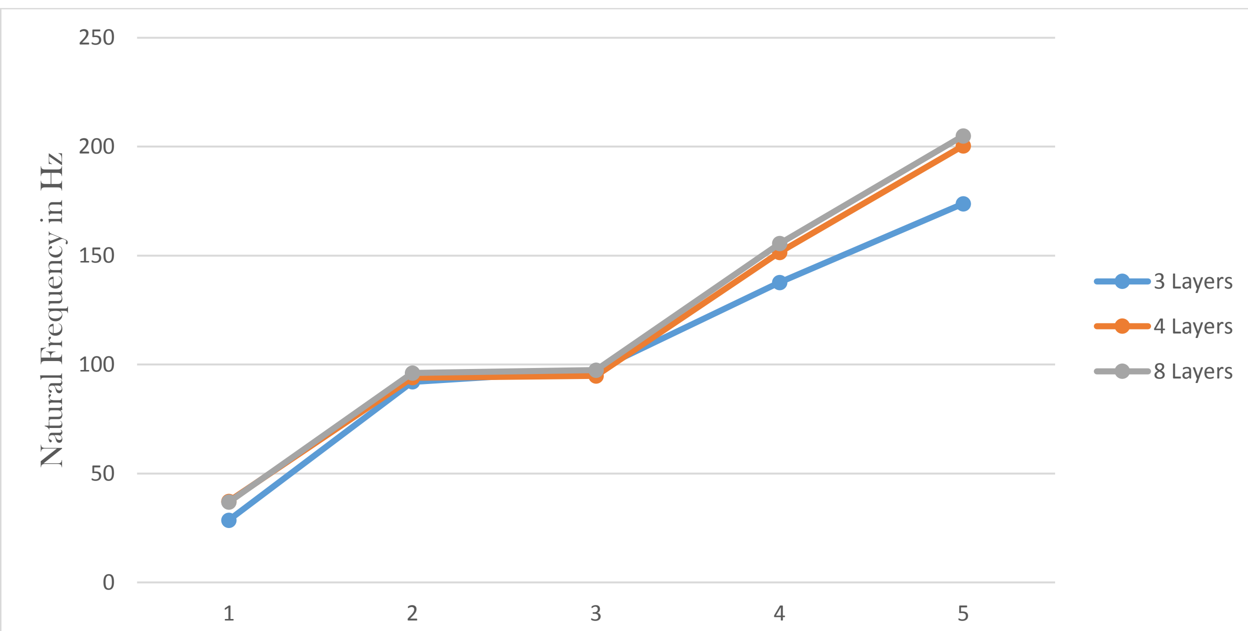



**Fig 5.3: Variation of frequency (Hz) with number of layers for SSSS square cross-ply with central circular hole**

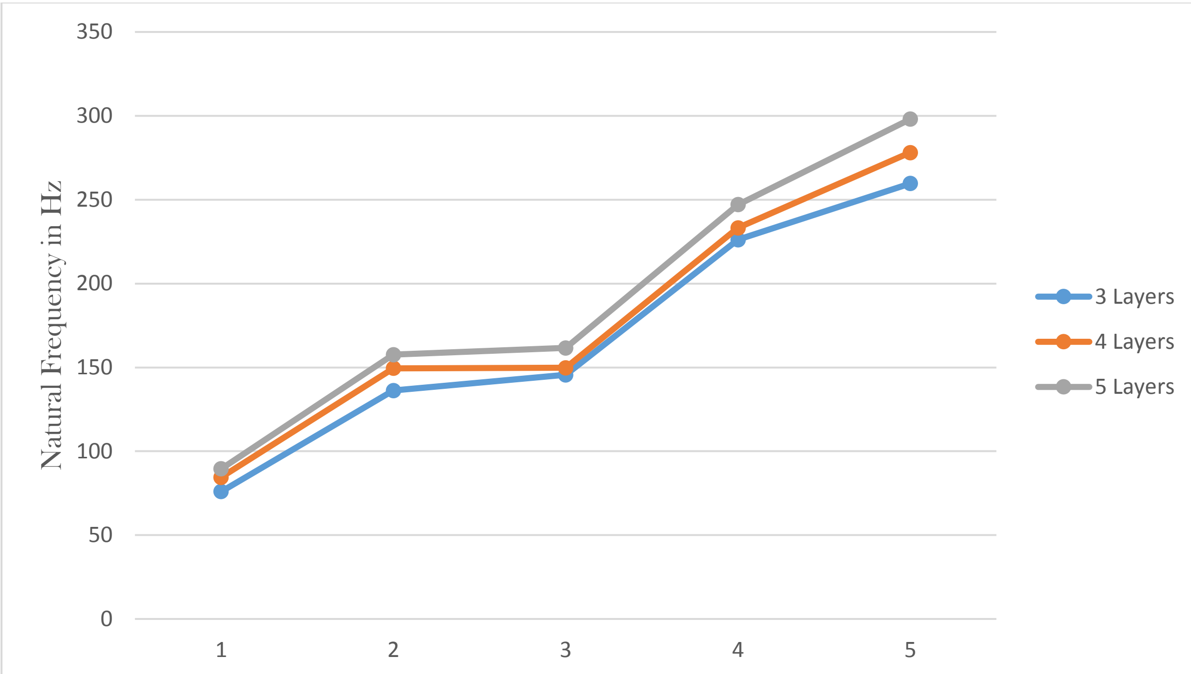



**Fig. 5.4: Variation of frequency (Hz) with number of layers for CCCC square cross-ply with central circular hole**

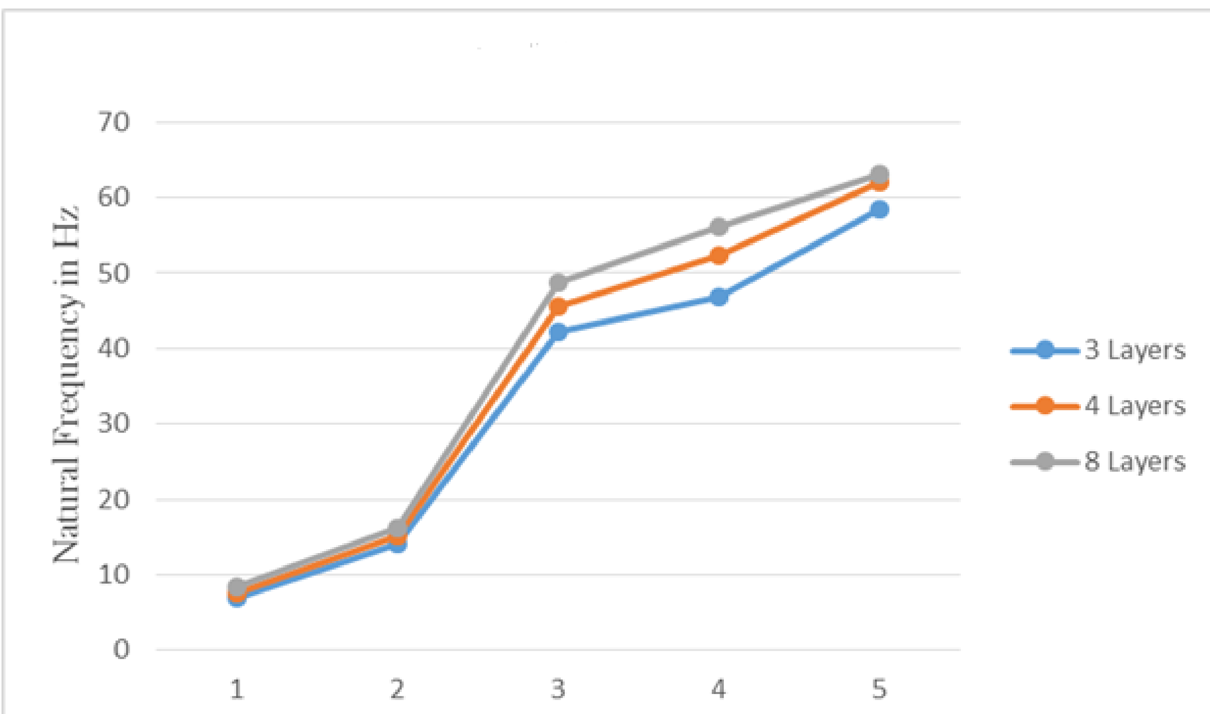
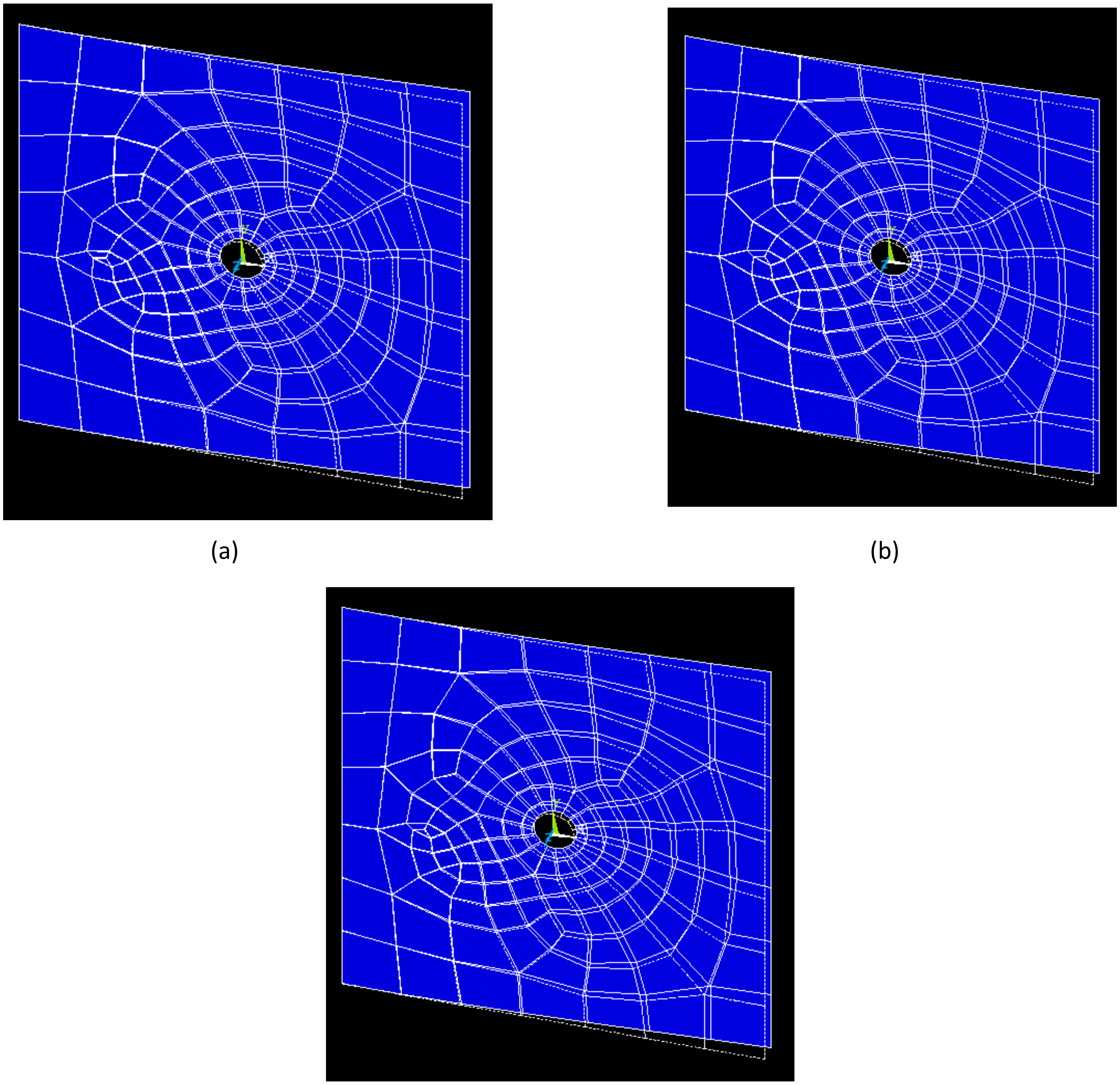



**Fig. 5.5: Variation of frequency (Hz) with number of layers for CFFF square cross-ply with central circular hole**



(a) (b) (c)

**Fig 5.6: The mode shapes for CFFF square ply for (a) 3 layers (0°/90°/0°), (b) 4 layers (0°/90°/90°/0°), (c) 8 layers (0/90/0/90/0/90/0/90)**

5.2.3 <u>Effect of Hole-size</u>

The above results were obtained for the hole-size $\left(\frac{D}{a}\right) = 0.1$. In this section, $\left(\frac{D}{a}\right)$ is varied from 0.1 to 0.5 and the first five natural frequencies are computed for a three layered simply supported cross-ply laminated plate.

**Table 9: First five natural frequencies (Hz) for SSSS square cross ply (0°/90°/0°) and hole-size: 0.1-0.5**

| Mode | $\left(\frac{D}{a}\right)=0.1$ | $\left(\frac{D}{a}\right)=0.2$ | $\left(\frac{D}{a}\right)=0.3$ | $\left(\frac{D}{a}\right)=0.4$ | $\left(\frac{D}{a}\right)=0.5$ |
|---|---|---|---|---|---|
| 1st | 28.53 | 29.92 | 30.29 | 31.12 | 32.54 |
| 2nd | 92.10 | 80.34 | 70.87 | 61.84 | 57.88 |
| 3rd | 97.30 | 83.83 | 82.97 | 68.26 | 68.65 |
| 4th | 137.66 | 135.76 | 132.71 | 118.44 | 107.68 |
| 5th | 173.68 | 166.80 | 163.23 | 137.96 | 133.42 |

From the above table, it can be concluded that the increase in hole-size increases the frequency slightly for the first mode of vibration. For higher modes, the value of natural frequency decreases with increasing hole-size. The following graph depicts the effect of hole size on the natural frequencies of vibration.

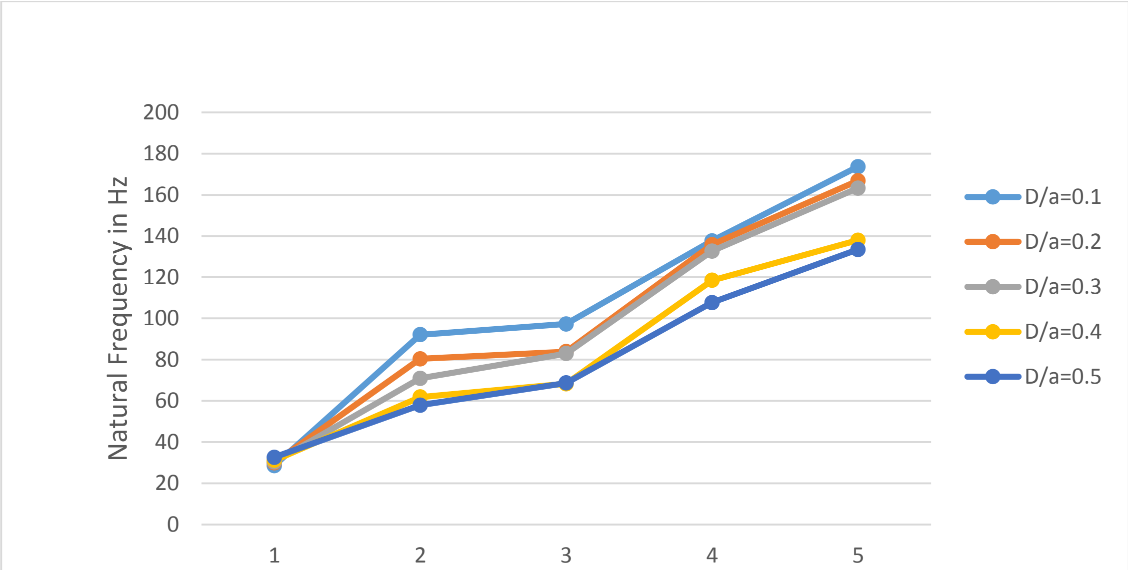



**Fig 5.7: Variation in natural frequencies (Hz) for SSSS square cross ply (0°/90°/0°) plate with varying hole-size**

5.2.4 Effect of Aspect Ratio

The aspect ratio is varied from 1 to 2. The width of the plate is fixed at $b = 0.5\,\text{m}$ and the length of the plate is varied. Accordingly, the diameter of the hole is also varied to satisfy $\left(\frac{D}{a}\right) = 0.2$ in each case. The first five natural frequencies for aspect-ratios 1, 1.5 and 2 are tabulated below.

**Table 10: First five natural frequencies for SSSS cross ply (0°/90°/0°) with varying aspect ratio**

| Aspect Ratio | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 |
|---|---|---|---|---|---|
| $\frac{a}{b} = 1$ | 29.92 | 80.34 | 83.83 | 135.76 | 166.80 |
| $\frac{a}{b} = 2$ | 20.43 | 32.81 | 53.85 | 62.84 | 83.52 |
| $\frac{a}{b} = 3$ | 20.45 | 25.42 | 37.93 | 45.30 | 63.44 |

It can be observed from the above table that the aspect ratio has relatively no effect on the first mode as compared to other modes. The natural frequencies for higher modes decrease with increasing aspect ratio. The following graph depicts the effect of aspect ratio on the natural frequencies of vibration.

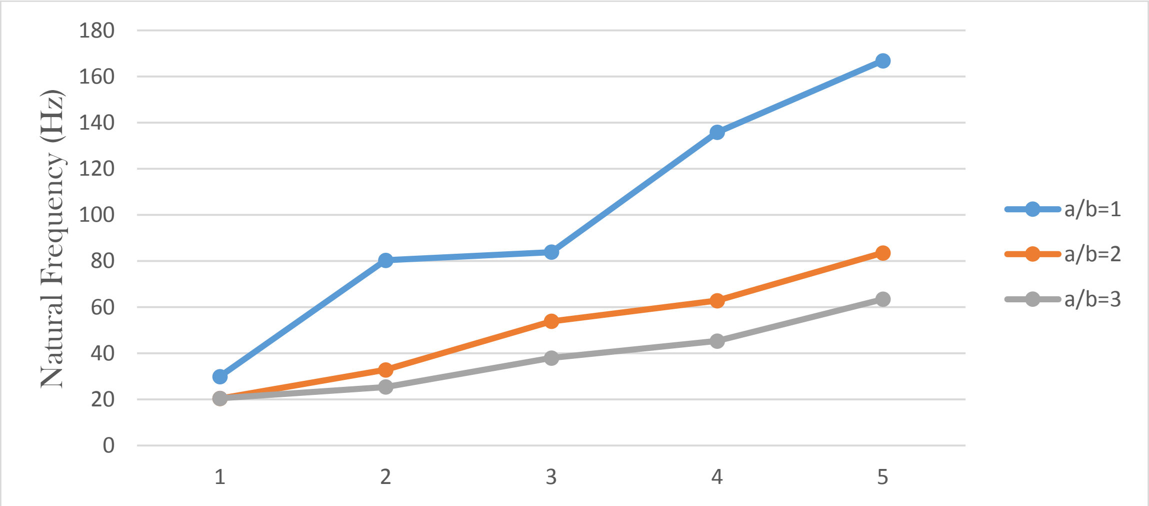



**Fig 5.8: Variation of natural frequencies for SSSS rectangular cross ply (0°/90°/0°) for varying aspect ratio**

5.2.5 Effect of fiber orientation

In this section, the first five natural frequencies are computed for different fiber orientations, ranging from $(15°/-15°/-15°/15°)$ to $(60°/-60°/-60°/60°)$. The values of natural frequencies for the first five modes are tabulated below.

**Table 11: First five natural frequencies (Hz) for SSSS square plate with a central circular hole for different fiber orientation**

| Fibre orientation | Mode 1 | Mode 2 | Mode 3 | Mode 4 | Mode 5 |
|---|---|---|---|---|---|
| $(15°/-15°/-15°/15°)$ | 41.82 | 109.87 | 116.83 | 182.15 | 211.66 |
| $(30°/-30°/-30°/30°)$ | 48.45 | 127.18 | 133.78 | 205.23 | 225.55 |
| $(45°/-45°/-45°/45°)$ | 54.74 | 136.57 | 140.33 | 221.31 | 244.58 |
| $(60°/-60°/-60°/60°)$ | 57.02 | 126.72 | 132.46 | 218.34 | 257.01 |

From the above table, it can be observed that the fundamental frequency of vibration increases with increase in angle of orientation of fibers.

5.2.6 Effect of Hole-shape

In this section, the natural frequencies of vibration are determined for a square cross ply (0°/90°/0°) with different hole shapes – circular, square, triangular, hexagonal. In all of the cases, the side of the hole $(\mathrm{D})$ bears a fixed ratio with the side of the square plate,i.e., $\left(\frac{\mathrm{D}}{\mathrm{a}}\right)=0.2$. The following table shows the first five natural frequencies for solid plate and plate with cutouts of different shapes.

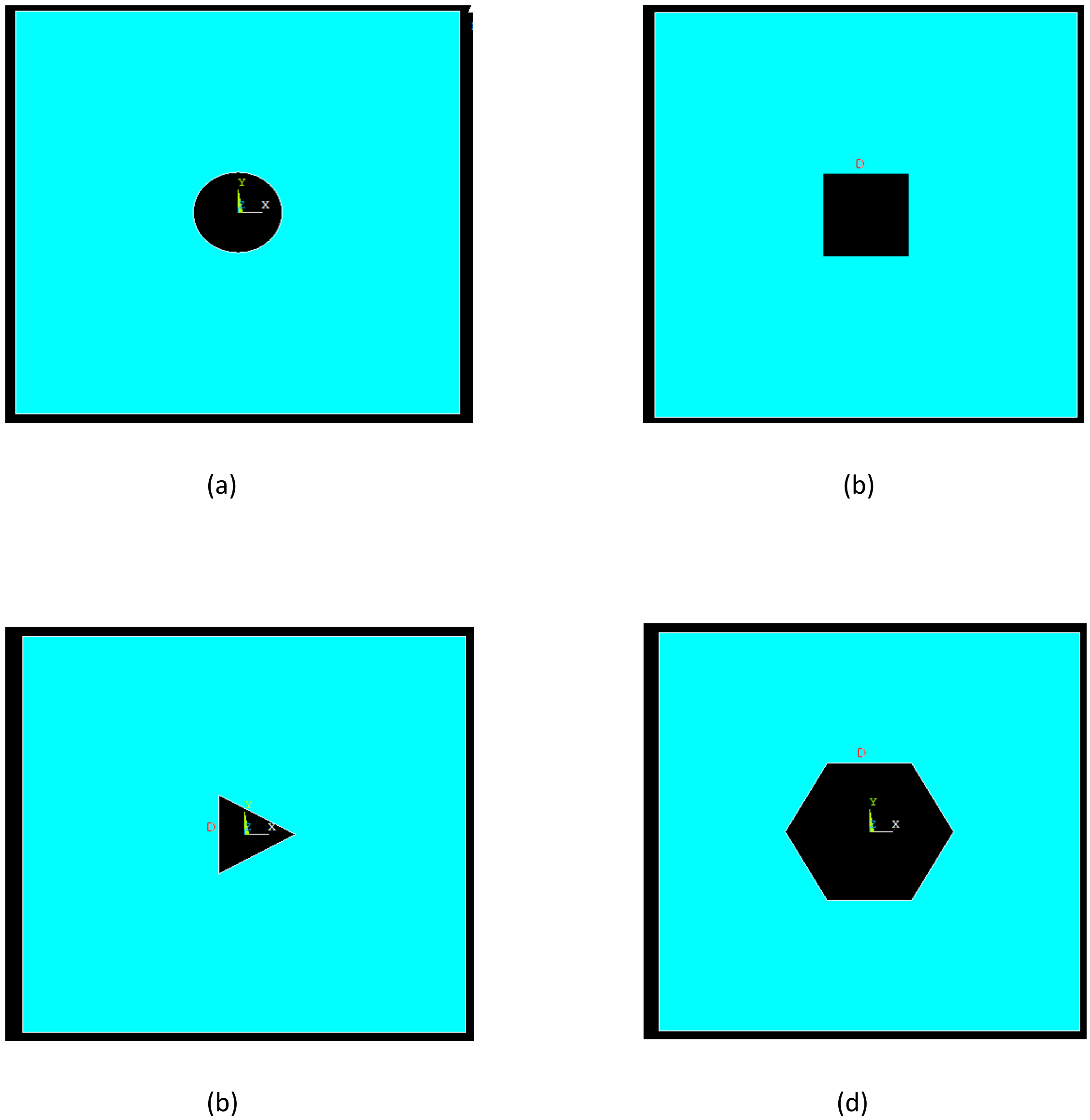



**Fig. 5.9: Square laminated composite plate with different shapes of cutouts – (a) circular, (b) square, (c) triangular, (d) hexagonal**

**Table 12: First five natural frequencies for SSSS square cross ply (0°/90°/0°) with cutouts of different shapes**

| Mode | Solid Plate | Circular Hole | Square Hole | Triangular Hole | Hexagonal Hole |
|---|---|---|---|---|---|
| 1 | 38.731 | 29.92 | 34.04 | 36.07 | 39.612 |
| 2 | 63.45 | 80.34 | 84.61 | 90.75 | 80.66 |
| 3 | 114.86 | 83.83 | 88.26 | 105.58 | 91.97 |
| 4 | 139.21 | 135.76 | 137.42 | 147.02 | 145.17 |
| 5 | 156.24 | 166.80 | 190.65 | 207.29 | 195.34 |

From the above table, it can be observed that the presence of cutouts of any shape decreases the value of natural frequencies. As can be seen, a plate with a triangular cutout shows relatively high frequencies and one with a circular cutout shows lower frequencies for any particular mode.

## 5.3 DISCUSSION:

Thus, the free vibration of a laminated composite plate is analyzed for different boundary conditions, different number of layers, hole-size, aspect ratio, fiber-orientation and hole shape.

- The frequency of a plate with cutout is reduced as compared to a solid plate owing to the reduction in its stiffness.
- The plate with cutout is found to be sensitive to boundary conditions applied. The plate showed highest frequencies when all of its outer edges were clamped. Lowest frequencies were obtained for the cantilever plate.
- With increasing number of layers of the laminated composite plate, the frequencies were observed to increase, irrespective of the boundary conditions applied.
- Similarly, the fundamental frequency increases with increasing the hole-size only for the first mode; but for higher modes, the values of natural frequencies decreased with increasing hole-size.

- With increase in the length of the plate, the values of natural frequencies were found to decrease for all modes.
- The fiber-orientation was varied from $(15°/-15°/-15°/15°)$ to $(60°/-60°/60°/-60°)$. The fundamental frequencies were found to increase with increase in angle of orientation of fibers.
- Four different shapes of cutouts – circular, square, triangular and hexagonal were considered and compared with a solid plate. The plate with a triangular cutout showed higher frequencies as compared to others and the plate with a circular cutout showed lowest frequencies.

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